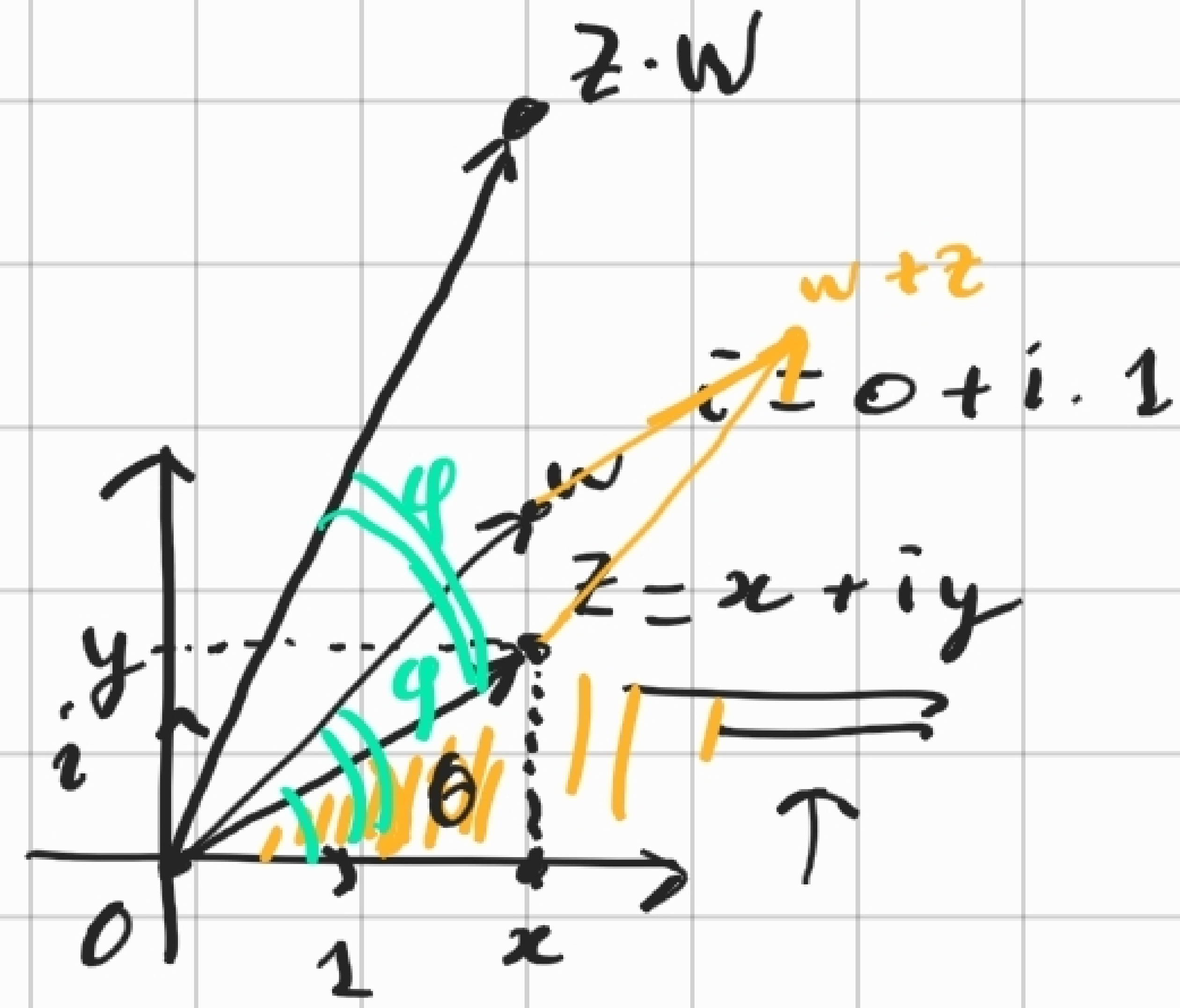
ANALISI MATEMATICA B

LEZIONE 36 – 16.12.2020

NUMERI COMPLESSI

$z \in \mathbb{C}$



Interpretazione geometrica del prodotto $z \cdot w$

$|z| = \sqrt{x^2+y^2} = \sqrt{z \cdot \bar{z}}$

$\bar{z} = x - iy$

$z \cdot \bar{z} = x^2 + y^2 = |z|^2$

$|z \cdot w| = |z| \cdot |w|$

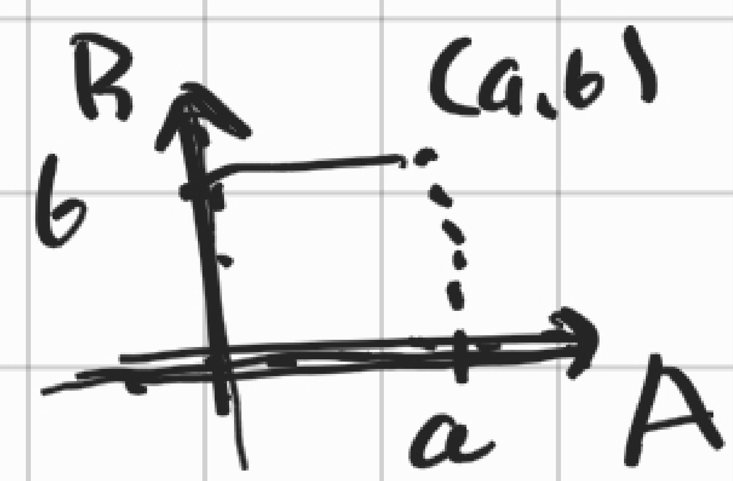
"$\theta = \operatorname{Arg} z$"

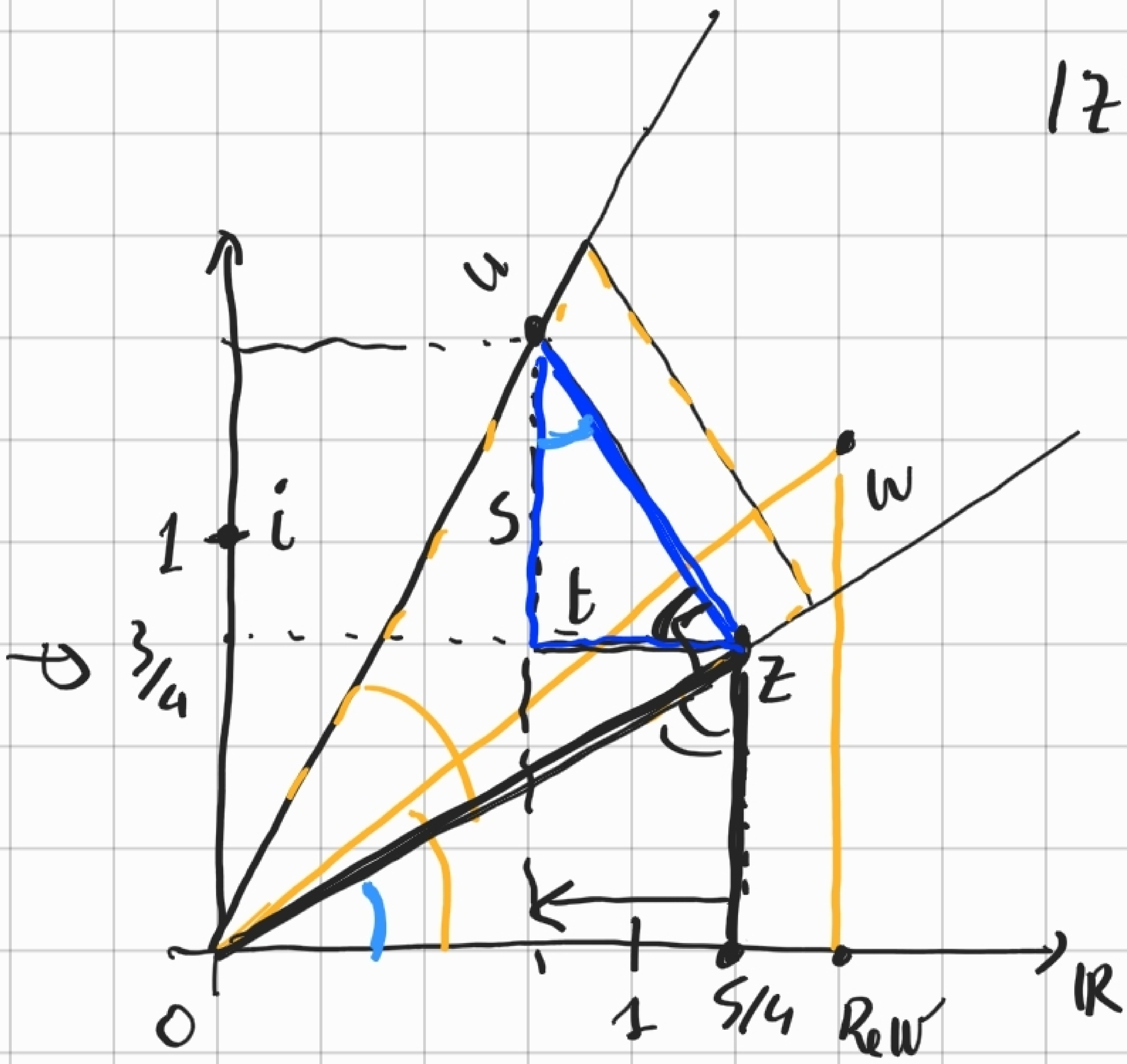
$\operatorname{Arg}(z \cdot w) = \operatorname{Arg} z + \operatorname{Arg} w$

$i^2 = -1$

Esempio $\quad z = \frac{5}{4} + \frac{3}{4} i \qquad w = \frac{6}{4} + \frac{5}{4} i$

A X B





$$|z| = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{3}{4}\right)^2} = \frac{\sqrt{25+9}}{4}$$

$$= \frac{\sqrt{34}}{4}$$

$$\mathrm{Re}\, w = \frac{3}{2}$$

$$|u - z| = \mathrm{Im}\, w \cdot \frac{|z|}{\mathrm{Re}\, w} = \frac{5}{4} \cdot \frac{\sqrt{34}}{4} \cdot \frac{2}{3} = \frac{5\sqrt{34}}{24}.$$

$$u = \mathrm{Re}\, z - t + i\left(\mathrm{Im}\, z + s\right)$$

$$\begin{cases} t = \dfrac{\mathrm{Im}\, z}{|z|} \, |u - z| = \dfrac{3}{\cancel{4}} \dfrac{\cancel{4}}{\cancel{\sqrt{34}}} \cdot \dfrac{5\cancel{\sqrt{34}}}{24} = \dfrac{5}{8} \\[2ex] s = \dfrac{\mathrm{Re}\, z}{|z|} \, |u - z| = \dfrac{5}{\cancel{4}} \cdot \dfrac{\cancel{4}}{\cancel{\sqrt{34}}} \cdot \dfrac{5\cancel{\sqrt{34}}}{24} = \dfrac{25}{24} \end{cases}$$

$$u = \frac{5}{4} - \frac{5}{8} + i\left(\frac{3}{4} + \frac{25}{24}\right)$$

$$= \frac{15}{24} + i\,\frac{43}{24}$$

$$z \cdot w = \left(\frac{5}{4} + \frac{3}{4} i\right) \cdot \left(\frac{6}{4} + \frac{5}{4} i\right)$$

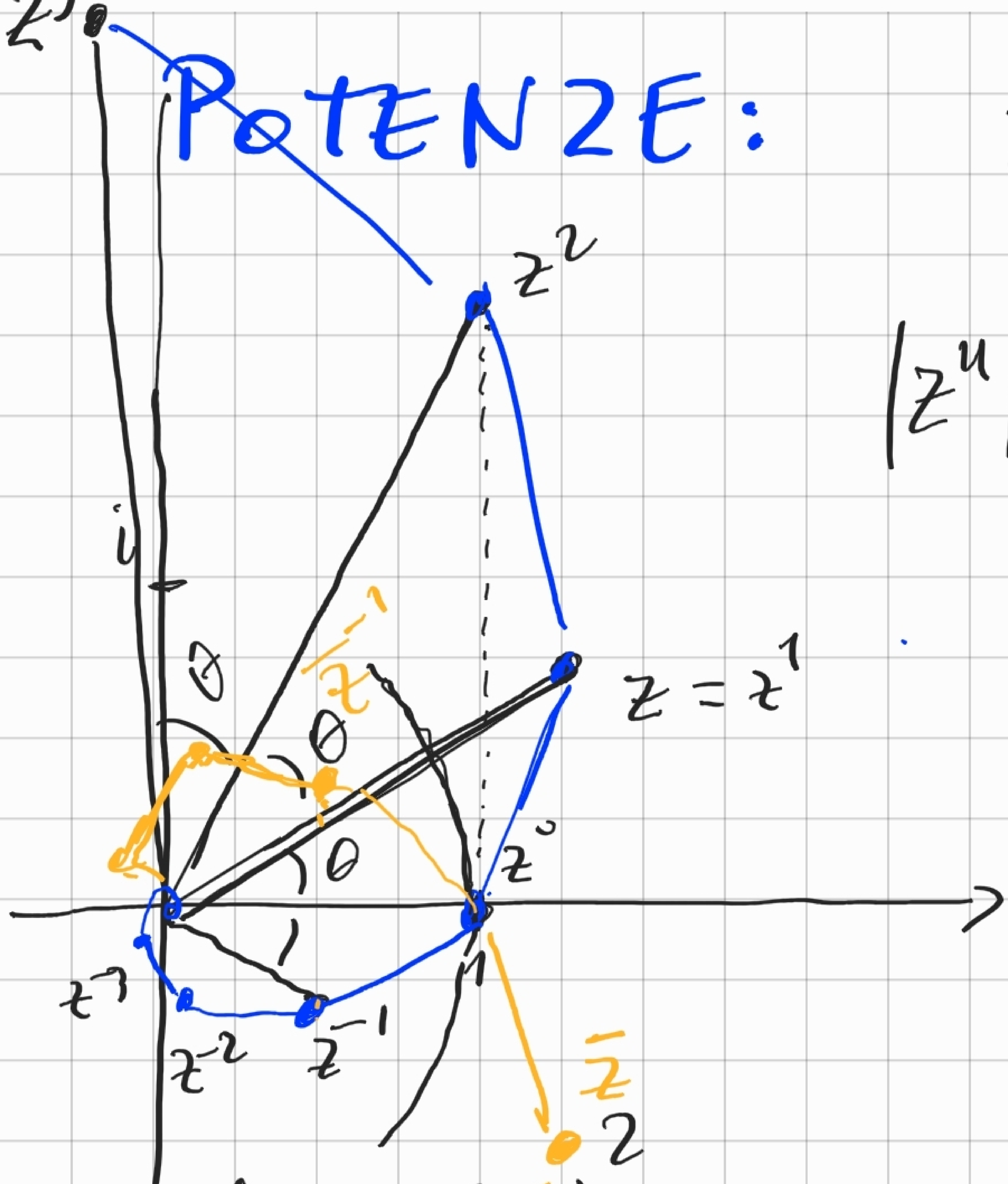
!!!

$$= \frac{30}{16} - \frac{15}{16} + i\left(\frac{18}{16} + \frac{25}{16}\right)$$

$$= \frac{15}{16} + i\,\frac{43}{16}$$

$$z \cdot w = \frac{24}{16} \cdot u \quad \checkmark$$

POTENZE: z^n



$|z^n| = |z|^n$

$$z^2 = \left(\frac{5}{4} + \frac{3}{4} i\right) = \frac{25}{16} - \frac{9}{16} + \frac{30}{16} i$$

$\bar{z}^3$ $\bar{z}^2$

$$= \frac{16}{16} + \frac{30}{16} i = 1 + \frac{15}{8} i$$

1 1
1 2 1
1 3 3 1

$$z^3 = \left(\frac{5}{4} + \frac{3}{4} i\right)^3$$

$$= \frac{5^3}{4^3} + 3\,\frac{5^2}{4^2} \cdot \frac{3}{4} i - \underbrace{3\,\frac{5}{4} \cdot \frac{9}{16}} - \frac{3^3}{4^3} i$$

$$= \frac{5^3 - 5 \cdot 27 + i\left(3^2 5^2 - 3^3\right)}{4^3}$$

$$= \frac{-10 + i\, 9 \cdot 22}{4^3} = \frac{-5}{32} + i\,\frac{9 \cdot 11}{32}$$

$$z^{-1} = \frac{1}{z} = \frac{1}{\frac{5}{4} + \frac{3}{4} i} = \frac{\frac{5}{4} - \frac{3}{4} i}{\frac{25}{16} + \frac{9}{16}}$$

$$= \frac{\frac{20}{16} - \frac{12}{16} i}{\frac{34}{16}} = \frac{20}{34} - \frac{12}{34} i$$

$$= \frac{10}{12} - \frac{6}{17} i = \frac{5}{6} - \frac{6}{17} i$$

Proprietà del coniugato:

$$\overline{w+z} = \bar{w} + \bar{z}$$

$$\bar{w}\cdot\bar{z} = \underbrace{\overline{(a+ib)}\,\overline{(x+iy)}}$$

$$= (a-ib)(x-iy)$$

$$= ax - by + i(-ay - bx) =$$

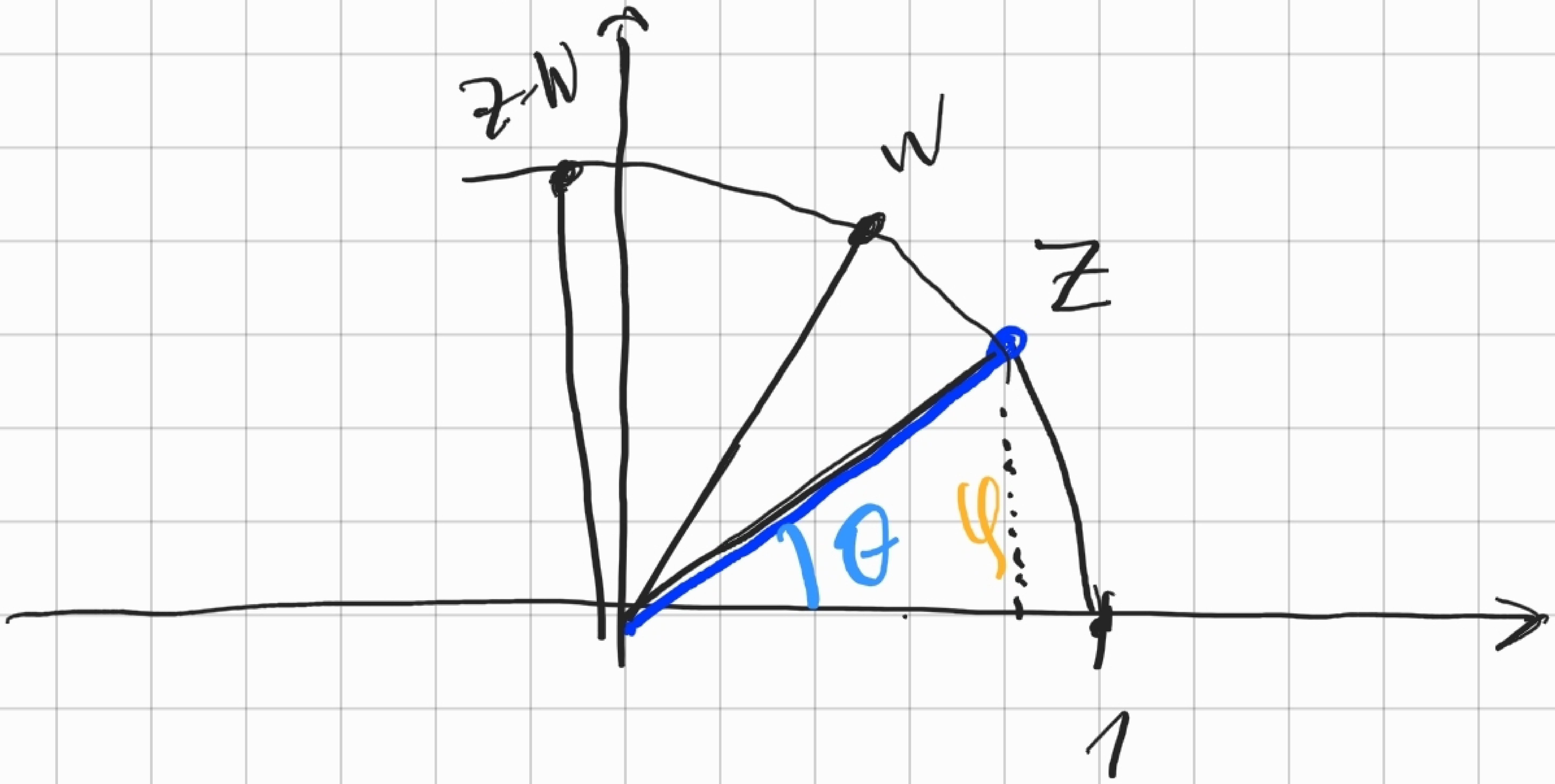
$$\overline{w\cdot z} = \overline{ax - by + i(bx + ay)}$$

$$= (ax-by) - i(bx + ay) =$$

$$\bar{z}^n = \overline{z^n}$$

COMPLESSI UNITARI

$|z| = 1$ (si dice che z è unitario)



$z = \frac{4}{5} + \frac{3}{5} i$ è unitario

$|z| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \frac{\sqrt{16+9}}{5} = 1.$

se $z = x + iy$ è unitario

$\arg z = \theta$

$$\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases} \qquad z = \cos\theta + i \operatorname{sen}\theta$$

se $|z| = 1$ e $|w| = 1$

$$\arg z = \theta \qquad \arg w = \varphi$$

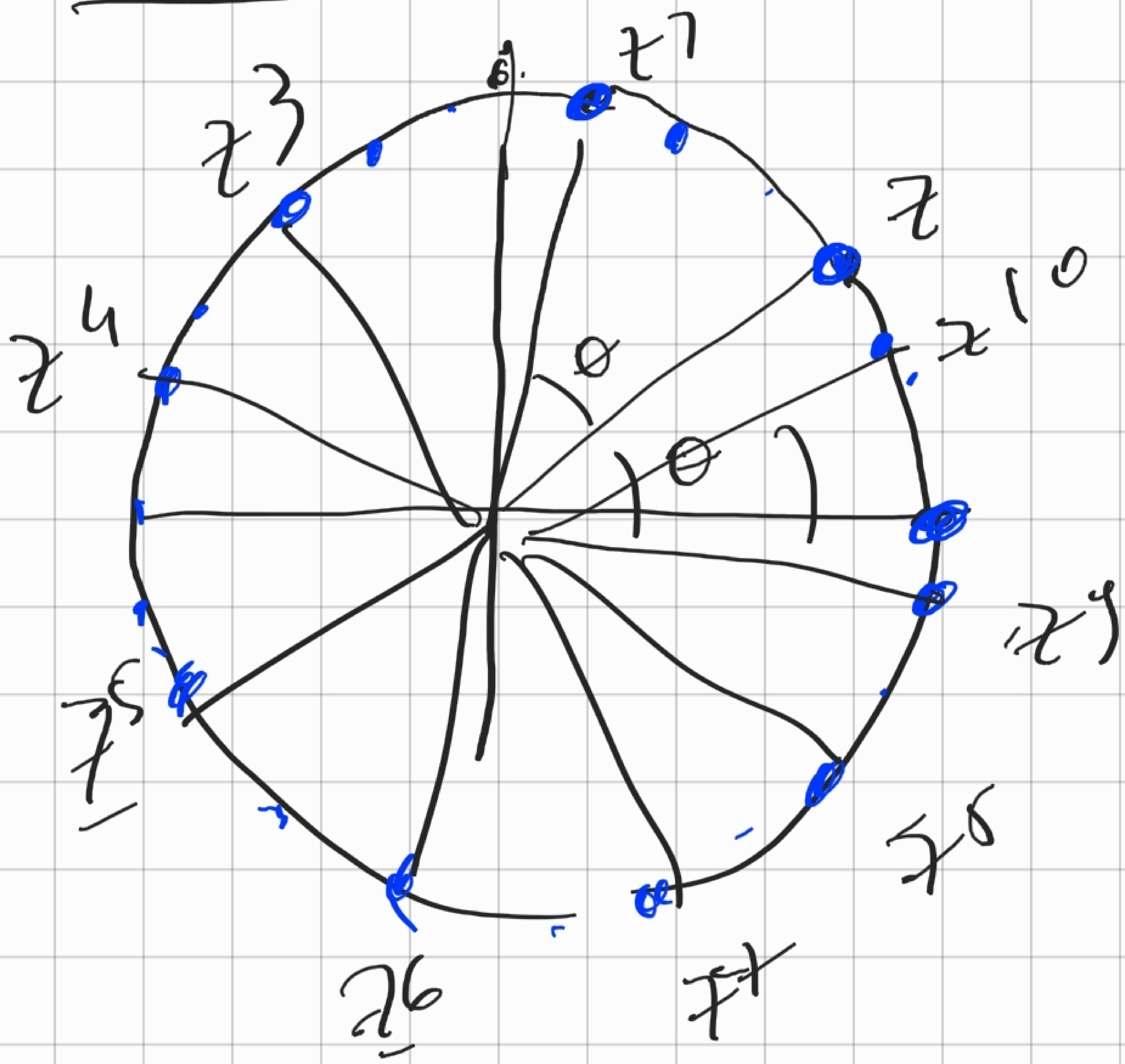
$$|z \cdot w| = 1$$

$$\arg(z \cdot w) = \arg z + \arg w$$

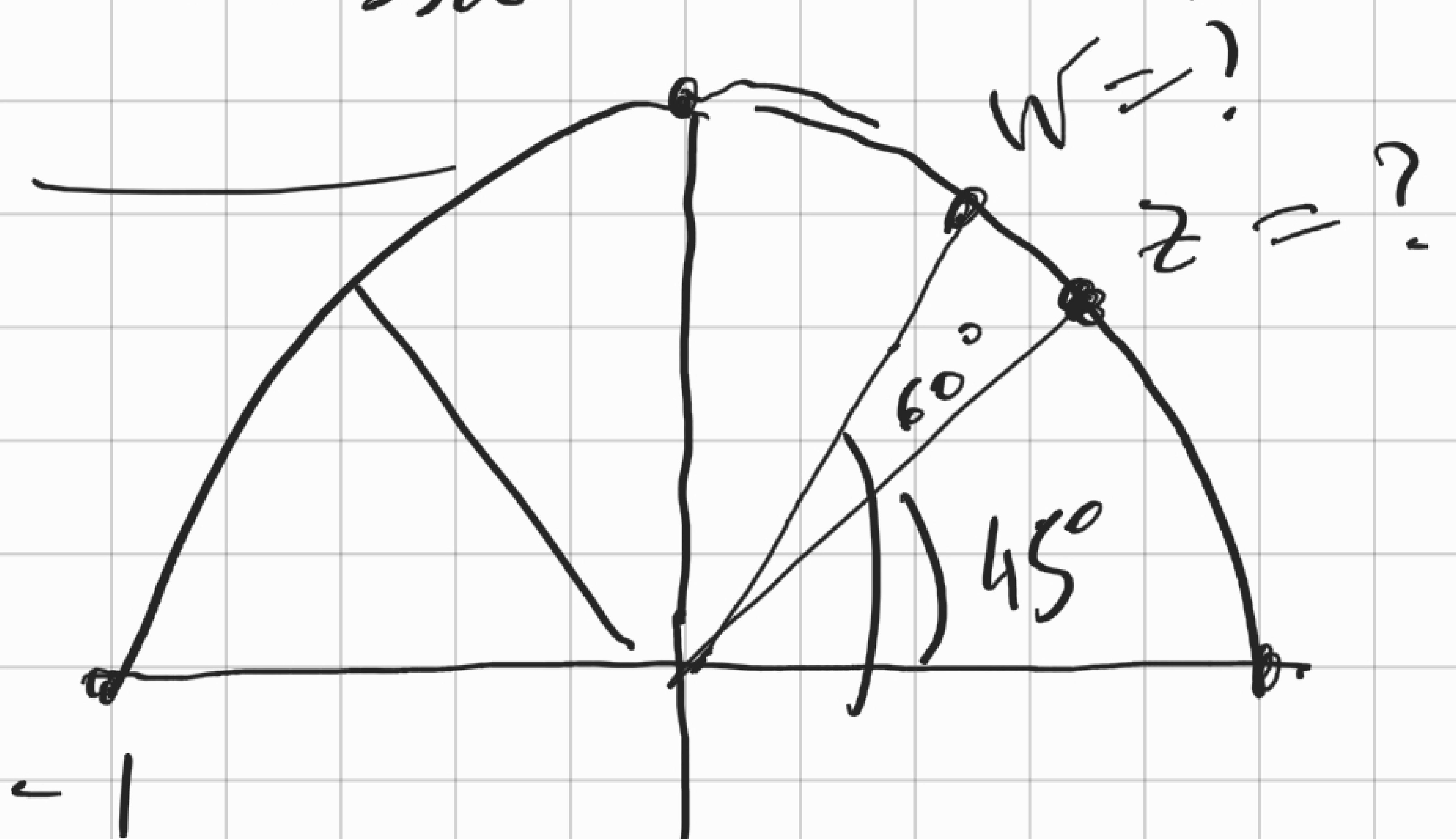
$$\begin{cases} z = \cos\theta + i \operatorname{sen}\theta \\ w = \cos\varphi + i \operatorname{sen}\varphi \end{cases}$$

$$\begin{aligned} z \cdot w &= \cos\theta\cos\varphi - \operatorname{sen}\theta \operatorname{sen}\varphi \\ &\quad + i\left(\cos\theta \operatorname{sen}\varphi + \sin\theta\cos\varphi\right) \\ &= \cos(\theta+\varphi) + i \operatorname{sen}(\theta+\varphi) \end{aligned}$$

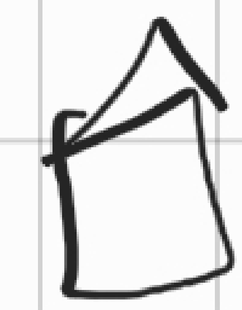
$$\begin{cases} \cos(\theta+\varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi \\ \sin(\theta+\varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi \end{cases}$$



Chissà se $\exists\, n : z^n = 1$?



$z^2 = i$ $\qquad$ $z = ?$



$\arg z = 45^\circ$

$\arg w = 60^\circ$

$w^3 = -1 \qquad w = a + ib$

$w^3 = (a+ib)^3 = a^3 + 3a^2 ib - 3ab^2 - ib^3$

$= a^3 - 3ab^2 + i(3a^2b - b^3)$

$= -1 = -1 + i(0)$

$$\begin{cases} a^3 - 3ab^2 = -1 \\ 3a^2b - b^3 = 0 \end{cases}$$

$b = 0$ ...

$b \neq 0 \quad \begin{cases} \\ 3a^2 = b^2 \end{cases}$

$$\begin{cases} b = \pm\sqrt{3}\,a \\ a^3 - 3a\,3a^2 = -1 \end{cases}$$

$$-8a^3 = -1$$

$$a^3 = \frac{1}{8} \qquad \begin{cases} a = \frac{1}{2} \\ b = \pm\frac{\sqrt{3}}{2} \end{cases}$$

$$w = \underset{\substack{\uparrow \\ \cos 60^\circ}}{\frac{1}{2}} + \underset{\substack{\uparrow \\ \sin 60^\circ}}{\frac{\sqrt{3}}{2}}\, i$$

$$\boxed{\sum \left(1+\frac{1}{n}\right)^n}$$

$$\frac{x^k}{\ln k}$$

"1^∞"

$a_n^{b_n}$

$\frac{1^k}{\ln k} \to 0$

$a_n \to 1$

$b_n \to +\infty$

$a_n \equiv 1 \qquad a_n^{b_n} = 1$

$$\left(1+\frac{1}{n}\right)^n$$

$a_n \to 0 \qquad a_n > 0$

$\frac{1}{a_n} \to +\infty$