LINEAR AND NONLINEAR WAVE PHENOMENA
10-14 SEPTEMBER, 2018

Organizers: F. Colombini, I. Gallagher, G. Staffilani, N. Visciglia

1. Program

MONDAY, 10/09/2018

9 : 20 – 9 : 30 Opening

9 : 30 – 10 : 10 N. Pavlovic: On well posedness for Boltzmann via dispersive tools

10 : 15 – 10 : 55 F. Merle: Inelasticity of soliton collisions for 5d critical NLW

10 : 55 – 11 : 25 **COFFEE-BREAK**

11 : 25 – 12 : 05 H. Koch: A family of conserved energies for Gross-Pitaevskii

LUNCH

16 : 00 – 16 : 40 M. Ifrim: A Morawetz inequality for water waves

16 : 40 – 17 : 10 **COFFEE-BREAK**

17 : 10 – 17 : 50 D. Tataru: Solitary waves in deep water

TUESDAY, 11/09/2018

9 : 30 – 10 : 10 N. Burq: Almost sure scattering for the one dimensional NLS

10 : 15 – 10 : 55 M. Berti: Long time dynamics of periodic water waves

10 : 55 – 11 : 25 **COFFEE-BREAK**

11 : 25 – 12 : 05 T. Nishitani: Transversally strictly hyperbolic systems

LUNCH

16 : 00 – 16 : 40 N. Tzvetkov: Solving the 4NLS with white noise initial data

16 : 40 – 17 : 10 **COFFEE-BREAK**

17 : 10 – 17 : 50 A. Parmeggiani: On the solvability of some degenerate operators

WEDNESDAY, 12/09/2018

9 : 30 – 10 : 10 O. Ivanovici: Dispersive estimates for the semiclassical Schrödinger equation inside a strictly convex domain
10 : 15 – 10 : 55 F. Planchon: Strichartz estimates for wave equation in strictly convex domains
10 : 55 – 11 : 25 **COFFEE-BREAK**
11 : 25 – 12 : 05 B. Pausader: TBA

LUNCH

20 : 00 **SOCIAL DINNER**

THURSDAY, 13/09/2018

9 : 30 – 10 : 10 P. Gérard: On the weakly damped Szegő equation
10 : 15 – 10 : 55 P. D'Ancona: On supercritical defocusing NLW outside a ball
10 : 55 – 11 : 25 **COFFEE-BREAK**
11 : 25 – 12 : 05 R. Carles: Universal dynamics for the logarithmic NLS

LUNCH

16 : 00 – 16 : 40 L. Vega: Evolution of polygonal lines by the binormal flow
16 : 40 – 17 : 10 **COFFEE-BREAK**
17 : 10 – 17 : 50 A. Nahmod: Global smooth flows for random SQG

FRIDAY, 14/10/2018

9 : 30 – 10 : 10 A. Maspero: Dynamics around finite gap sol. of NLS on $\mathbb{T}^2$
10 : 15 – 10 : 55 V. Petkov: NLW with time periodic potential
10 : 55 – 11 : 25 **COFFEE-BREAK**
11 : 25 – 12 : 05 J.-M. Bony: On the relations between F.I.O. and their symbols

LUNCH

2. Abstracts

- **N. Burq**: TBA
- **M. Berti**: In this talk I shall present recent results about the complex dynamics of the water waves equations of a bi-dimensional fluid under the action of gravity and/or capillary forces, with space periodic boundary conditions. We shall discuss both Birkhoff normal form long time existence results as well as KAM theorems about the bifurcation of small amplitude quasi-periodic solutions.
- **J.-M. Bony**: The principal symbol of a Fourier Integral Operator, in the case of homogeneous phase functions, is defined by Hörmander in 1971 as a section of a line bundle, called the Maslov bundle. This bundle is abstractly defined via its transition functions and has no concrete meaning. More recently, I developed a theory of Fourier Integral Operator associated to canonical transformations which are not supposed homogeneous. It becomes natural to define their symbol as sections of a more concrete bundle, made of metaplectic operators. I shall explain the links between the two points of view, which lead to a concrete definition of the Maslov bundle in the case of (almost) homogeneous phase functions.
- **R. Carles**: We consider the nonlinear Schrodinger equation with a logarithmic nonlinearity, whose sign is such that no non-trivial stationary solution exists. Explicit computations show that in the case of Gaussian initial data, the presence of the nonlinearity affects the large time behaviour of the solution, on at least three aspects. The dispersion is faster than usual by a logarithmic factor in time. The positive Sobolev norms of the solution grow logarithmically in time. Finally, after rescaling in space by the dispersion rate, the modulus of the solution converges to a universal Gaussian profile (whose variance is independent of the initial variance). In the case of general initial data, we show that these properties remain, up to weakening the third point (weak convergence instead of strong convergence). One of the key steps of the proof for the last point consists in using a fluid formulation. It reduces the equation to a variant of the isothermal compressible Euler equation, whose large time behaviour turns out to be governed by a parabolic equation involving a Fokker-Planck operator.
- **P. D'Ancona**: In this work I consider a defocusing semilinear wave equation, with a power nonlinearity, defined on the outside of the unit ball of Rn, and with Dirichlet conditions at the boundary. The power is assumed to be sufficiently large, $p > O(n)$, and the space dimension is 3 or larger. Even in the radial case, the corresponding problem on Rn is completely open. Here I construct a family of large global solutions, whose data are small perturbations of radial initial data in suitable weighted Sobolev norms of higher order.
- **P. Gérard**: The Szego equation is an integrable model for lack of dispersion on the circle. An important feature of this model is the existence of a residual set — in the Baire sense— of initial data leading to superpolynomially unbounded trajectories in high Sobolev norms. It is therefore natural to study the effect of a weak damping on such a system. In this talk I will discuss the damping of the lowest Fourier mode, which presents the advantage of keeping part of the integrable structure. Somewhat surprinsingly,

we shall show that such a weak damping leads to a wider set of unbounded trajectories in high Sobolev norms. This is a joint work in collaboration with Sandrine Grellier.

- **M. Ifrim**: We consider gravity and gravity/capillary water waves in two space dimensions. Assuming uniform energy bounds for the solutions, we prove local energy decay estimates. Our result is uniform in the infinite depth limit.
- **O.Ivanovici**: We consider the semiclassical Schrdinger equation inside a strictly convex domain and we obtain dispersion estimates for the linear flow; we highlight a 1/4 loss compared to the flat case.
- **H. Koch**: A difficulty in the study of the Gross-Pitaevskii equation is that the state space is nonlinear. In joint work with Xian Liao we equip it with a new metric, and construct a continuous family of conserved energies.
- **A. Maspero**: TBA
- **F. Merle**: TBA
- **A. Nahmod**: In this talk we consider the surface quasi-geostrophic equation (SQG) where randomness is injected into the system via a random diffusion term. We are interested in the question of whether and how randomness restores some form of uniqueness. We show the existence of pathwise unique global solutions, thus exhibiting an instance of 'regularization by noise'. This is joint work with Buckmaster-Staffilani and Widmayer.
- **T. Nishitani**: We consider the Cauchy problem for first-order systems. Assuming that the characteristic variety is a smooth manifold and the characteristic values are real and semisimple we introduce a new class which is strictly hyperbolic in the directions transverse to the characteristic manifold. If the propagation cone and the characteristic manifold are compatible we prove that transversally strictly hyperbolic systems are strongly hyperbolic systems. On the other hand if the propagation cone is incompatible with the characteristic manifold then transversally strictly hyperbolic systems are much more involved. In this talk we mainly discuss this case taking an interesting example proposed by G.Métivier.
- **A. Parmeggiani**: In this talk, I will be describing some recent results obtained in joined work with Serena Federico, regarding the local solvability of degenerate oper- ators of the kind

$$\sum_{j=1}^{N} X_j^* f_j X_j + X_{N+1} + iX_0$$

  in various situations. The X_j, X_{N+1} and X_0 are first-order differential operators, with homogeneous symbol of degree 1, where X_{N+1} and X_0 have a real symbol, and the f_j are smooth real-valued functions. This class (which contains operators that are non-hypoelliptic) is a generalization of Kannais celebrated example and that studied by Colombini, Cordaro and Pernazza. We shall give sufficient conditions which ensure the local solvability of P. In general, one will have L^2 to L^2 local solvability.
- **N. Pavlovic**: Boltzmann equation is a partial differential equation which describes the evolution of the probability density of independent identically distributed particles modeling a rarefied gas with predominantly binary elastic interactions. In this talk we will focus on local theory of

well-posedness. Our main intention is not to investigate optimal regularity spaces for solving Boltzmann equation. Rather, we demonstrate the close connection between Boltzmann equation and nonlinear Schrödinger equations in the density matrix formulation; this connection has been recognized implicitly for some time, but we wish to make it quite explicit and to the best of our knowledge this is the first time such an explicit connection has been established. The talk is based on joint works with Thomas Chen and Ryan Denlinger.

- **B. Pausader**: TBA
- **F. Planchon**: We prove sharper Strichartz estimates than expected from the optimal dispersion bounds. This follows from taking full advantage of the space-time localization of caustics. Several improvements on the parametrix construction are obtained along the way and are of independent interest. Moreover, we extend the range of known counterexamples by propagating carefully constructed Gaussian beams, proving that our Strichartz estimates are sharp in some regions of phase space. This is joint work with O.Ivanovici and G.Lebeau.
- **V. Petkov**:It is known that for some time periodic potentials $q(t,x) \geq 0$ having compact support with respect to x some solutions of the Cauchy problem for the wave equation $\partial_t^2 u - \Delta_x u + q(t,x)u = 0$ have exponentially increasing energy as $t \to \infty$. We show that if one adds a nonlinear defocusing interaction $|u|^r u, 2 \leq r < 4$, then the solution of the nonlinear wave equation exists for all $t \in \mathbb{R}$ and its energy is polynomially bounded as $t \to \infty$ for every choice of q. Moreover, we prove that the zero solution of the nonlinear wave equation is instable if the corresponding linear equation has the property mentioned above. This is a joint work with N. Tzvetkov.
- **D. Tataru**: Solitary waves are waves on the surface of the water which keep a constant profile and which move with constant velocity. Two longstanding open problems have been whether such waves exist in deep water in the presence of either gravity or surface tension, but not both. This talk will provide the answers to both of these problems in two space dimensions. This is joint work with Mihaela Ifrim.
- **N. Tzvetkov**: We will consider the fourth order Nonlinear Schroedinger equation, posed on the circle, with initial data distributed according to the white noise. This problem is well posed for smooth initial data. It is therefore natural to consider the sequence of smooth solutions with data distributed according regularisations (by convolution) of the white noise. We show that a renormalisation of this sequence converges to a unique limit. The limit has the white noise as an invariant measure. The proof shares some features with the modified scattering theory which received a lot of attention in the PDE community. As a consequence the solution has a more intricate singular part compared to the large body of literature on probabilistic well-posedness for dispersive PDE's. This is a joint work with Tadahiro Oh and Yuzhao Wang.
- **Vega**: The aim of the talk is threefold. First we display solutions of the cubic nonlinear Schrödinger equation on $\mathbb{R}$ in link with initial data a sum of Dirac masses. Secondly we show a Talbot effect for the same equation. Finally we prove the existence of a unique solution of the binormal flow

with datum a polygonal line. This equation is used as a model for the vortex filaments dynamics in 3-D fluids and superfluids. We also construct solutions of the binormal flow that present an intermittency phenomena.