$f(x,y)$, $\exists \nabla f(x_0,y_0)$?

$$\underbrace{\frac{\partial f}{\partial x}(x,y)} \qquad (x,y) \neq (x_0,y_0)$$

$$\lim_{(x,y)\to(x_0,y_0)} \frac{\partial f}{\partial x}(x,y) = \underbrace{\frac{\partial f}{\partial x}(x_0,y_0)}$$

$$\lim_{(x,y)\to(x_0,y_0)} \frac{\partial f}{\partial x}(x,y) = \underbrace{L} \leftarrow (-\infty,\infty)$$

$$\lim_{(x,y)\to(0,0)} \frac{[\operatorname{sen}(xy) - xy]}{x^2+y^2} = \frac{-\frac{x^3y^3}{6} + o(x^3y^3)}{x^2+y^2}$$

$$\operatorname{sen} t = t - \frac{t^3}{6} + o(t^3), \quad \boxed{t \to 0}$$

$$\operatorname{sen} t - t = -\frac{t^3}{6} + o(t^3)$$

$$\boxed{xy = t} \to 0$$

$$\operatorname{sen}(xy) - xy = -\frac{x^3y^3}{6} + o(x^3y^3)$$

$$\underbrace{\lim_{(x,y)\to(0,0)} -\frac{x^3y^3}{6(x^2+y^2)}}_{\boxed{l_1 = 0}} + \underbrace{\lim_{(x,y)\to(0,0)} \frac{o(x^3y^3)}{x^2+y^2}}_{\boxed{l_2 = 0}}$$

$$-\frac{\rho^4 \sin^3\alpha \cos^3\alpha}{6}$$

$$\left|\frac{\rho^4 \sin^3\alpha \cos^3\alpha}{6}\right| \leq \frac{\rho^4}{6} \xrightarrow{\rho\to 0} 0$$

$$\frac{o(x^3y^3)}{x^2+y^2} \xrightarrow{(x,y)\to(0,0)} \boxed{?}$$

(Hp) $$\frac{o(x^3y^3)}{x^3y^3} \xrightarrow{(x,y)\to(0,0)} 0$$

(Ts) $$\boxed{\frac{o(x^3y^3)}{x^2+y^2}} \xrightarrow{(x,y)\to(0,0)} 0$$

Dim. $$\boxed{\underbrace{\frac{o(x^3y^3)}{x^3y^3}}_{\downarrow\,(Hp)\;0}\cdot\underbrace{\left(\frac{x^3y^3}{x^2+y^2}\right)}_{\downarrow\;0}} \longrightarrow 0$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy) - xy}{x^2+y^2}$$

$$\boxed{\frac{\sin(xy)}{x^2+y^2}} - \frac{xy}{x^2+y^2}$$

N.E. $\quad \ell_1 \quad \ell_2 \quad$ N.E.

$$\frac{\operatorname{sen}(xy)}{x^2+y^2} = \underbrace{\underbrace{\frac{\operatorname{sen}(xy)}{xy}}_{\downarrow\ 1} \cdot \underbrace{\frac{xy}{x^2+y^2}}_{\downarrow\ \text{N.E.}}}_{\Downarrow\ \text{N.E.}}$$

$$\operatorname{sen} t = t + o(t)$$

$$\Downarrow$$

$$\operatorname{sen}(xy) = xy + o(xy) \implies \frac{\operatorname{sen}(xy) - xy}{o(xy)} =$$

$$\lim_{(x,y)\to(0,0)} \boxed{\frac{o(xy)}{\underset{\neq 0}{x^2+y^2}}} \overset{?}{=} 0$$

$$2|x||y| \leq x^2+y^2$$

$$\Downarrow$$

$$\frac{2|x||y|}{x^2+y^2} \leq 1$$

(Hp) $\quad \dfrac{o(xy)}{xy} \longrightarrow 0$

(Tn) $\quad \dfrac{o(xy)}{\underset{\neq 0}{x^2+y^2}} \overset{?}{\longrightarrow} 0\ ?$

<u>DIM.</u>

$$\frac{o(xy)}{\underset{\neq 0}{x^2+y^2}} = \underbrace{\frac{o(xy)}{xy}}_{\downarrow\ 0} \cdot \underbrace{\left|\frac{xy}{\underset{\neq 0}{(x^2+y^2)}}\right|}_{\leq\ \frac{1}{2}} \longrightarrow 0$$

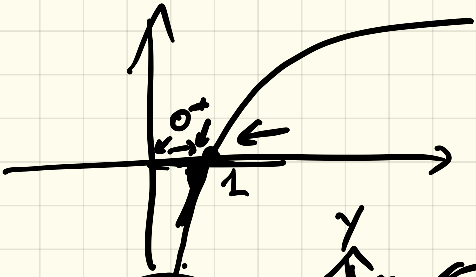
con = norme = limitate!

$$\lim_{(x,y)\to(0,0)} \frac{\ln(|xy|)}{\ln(1-x^2-y^2)}$$

$\ln(|xy|) \to -\infty$

$= \boxed{+\infty}$

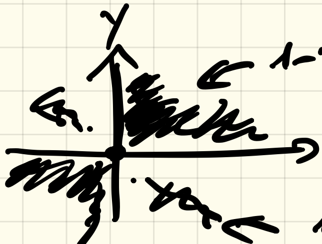
$\ln(1-x^2-y^2) \to \ln 1 = 0^-$



$-\infty$

$$\nexists \lim \frac{\ln(|xy|)}{\ln(1-xy)}$$

$\ln(|xy|) \to -\infty$



$1-xy \to 1^-$

$1-xy \to 1^+$

$\ln(1-xy) \to \boxed{0}^+$ se $xy<0$

0^- se $xy>0$

$$\lim_{(x,y)\to(0,0)} \frac{\ln\left(1+(x+y)\right)-(x+y)}{x^2+y^2}$$

$$\ln\left(1+(x+y)\right)-(x+y) \to \ln 1 = \boxed{0}$$

$$x^2+y^2 \to \boxed{0}$$

$$\frac{\ln\left(1+\rho(\cos\theta+\sin\theta)\right)-\rho(\cos\theta+\sin\theta)}{\rho^2}$$

$$\ln(1+t) = \underbrace{t-\frac{t^2}{2}+o(t^2)}$$

$$t = \underbrace{(x+y)}_{\to 0}, \quad t \to 0$$

$$\ln\left(1+(x+y)\right) = \underbrace{\boxed{x+y}-\frac{(x+y)^2}{2}+o\left((x+y)^2\right)}$$

$$\lim_{(x,y)\to(0,0)} \frac{-\frac{(x+y)^2}{2} + o\left((x+y)^2\right)}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} -\frac{(x+y)^2}{2(x^2+y^2)} \longrightarrow l_1 \quad \boxed{N.E.}$$

$$\lim_{(x,y)\to(0,0)} \frac{o\left((x+y)^2\right)}{x^2+y^2} \longrightarrow \boxed{l_2} = 0$$

$$\lim_{(x,y)\to(0,0)} -\frac{(x+y)^2}{2(x^2+y^2)} =$$

$$= \lim_{(x,y)\to(0,0)} \frac{-(x^2+y^2+2xy)}{2(x^2+y^2)} =$$

$$= \lim_{(x,y)\to(0,0)} \left[\underbrace{-\frac{\cancel{(x^2+y^2)}}{2\cancel{(x^2+y^2)}}}_{\downarrow \\ -\frac{1}{2}} - \underbrace{\frac{\cancel{2}xy}{\cancel{2}(x^2+y^2)}}_{\downarrow \\ \boxed{N.E.}} \right]$$

$$\boxed{ab \leq \frac{a^2}{2} + \frac{b^2}{2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{o\left((x+y)^2\right)}{x^2+y^2}$$

limitata

$$\frac{o\left((x+y)^2\right)}{(x^2+y^2)} = \frac{o\left((x+y)^2\right)}{(x+y)^2} \cdot \left[\frac{(x+y)^2}{(x^2+y^2)}\right]$$

$\downarrow$

0

$$\frac{x^2+y^2+2xy}{x^2+y^2} = \left(|1| + \left|\frac{2xy}{x^2+y^2}\right|\right) \leq 2$$

$\leq 1 \qquad \leq 1$

$$\ln(1+t) = t + o(t)$$

$$\ln(1+(x+y)) - (x+y) = o(x+y)$$

$$\boxed{\lim_{(x,y)\to(0,0)} \left[\frac{o(x+y)}{x^2+y^2}\right]} = ?$$

$$\frac{o(x+y)}{(x+y)} \cdot \boxed{\frac{x+y}{x^2+y^2}}$$

$\downarrow$ 0 $\qquad$ $\downarrow$ non è limitata

$y=0$ e x libera

$$\frac{x}{x^2} = \frac{1}{x}$$

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos^4(xy)}{\operatorname{sen}(x^4+y^4)} = N.E.$$

$$\operatorname{sen}(x^4+y^4) \sim x^4+y^4$$

$$\boxed{\frac{1-\cos^4(xy)}{x^4+y^4}} \qquad \frac{x^4+y^4}{\operatorname{sen}(x^4+y^4)} \to 1$$

$\rightarrow 0$

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos^4(xy)}{x^4+y^4} =$$

$\rightarrow 0$ $\rightarrow 2$

$$= \frac{(1-\cos^2(xy))(1+\cos^2(xy))}{x^4+y^4}$$

$\rightarrow$ N.E.

$$\frac{1-\cos^2(xy)}{x^4+y^4} =$$

$$= \frac{\operatorname{sen}^2(xy)}{x^4+y^4}$$

N.E.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2(x-y)}{(x^4+y^4)} = \text{N.E.}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy^2(x-y)}{x^2y^2} \; \frac{x^2y^2}{x^4+y^4}$$

$$\frac{xy^2(x-y)}{x^2y^2} \to 1 \qquad \frac{x^2y^2}{x^4+y^4} \to \text{N.E.}$$

$$\frac{xy}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2y^2}{x^4+y^4}\right) \to \text{N.E.}$$

$Df(0,0)$

$f(x,y) = |x^2 + 2y|$

$$\frac{\partial}{\partial x} f(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} =$$

$$= \lim_{h \to 0} \frac{|h^2|}{h} = \boxed{0}$$

$$\partial_y f(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} =$$

$$= \lim_{k\to 0} \frac{|\operatorname{sen} k|}{k}$$ N.E.

$$\nabla f(0,0)$$ N.E.

$$\lim_{k\to 0^+} \frac{|\operatorname{sen} k|}{k} = \lim_{k\to 0^+} \frac{\operatorname{sen} k}{k} = 1$$

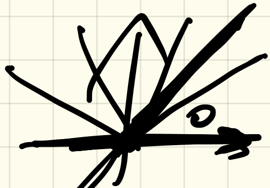
$$\lim_{k\to 0^-} \frac{|\operatorname{sen} k|}{k} = \lim_{k\to 0^+} - \frac{\operatorname{sen} k}{k} = -1$$

$$\lim_{(x,y)\to(0,0)} \frac{\ln(x^2+y^2)}{\ln(x^2+2y^2)} =$$

$$= \frac{\ln(x^2+y^2)}{2(x^2y^2)} =$$

$$= \frac{1}{2}\left[\frac{2(x^2+y^2)}{2|xy|}\right] \longrightarrow \text{N.E.}$$

$$2\left(\frac{x^2y^2}{x^4+y^4}\right) \quad \text{N.E.}$$



$$2\,\frac{\cancel{\rho^4}\,\overset{1}{\cos^2\theta}\;\overset{0}{\sin^2\theta}}{\cancel{\rho^4}\left(\cos^4\theta+\sin^4\theta\right)} \to \frac{1}{2}$$

$$\to \boxed{f(\theta)} \neq 0$$

$$2\,\frac{1}{(\sqrt{2})^2}$$

$x^2 = X$

$y^2 = Y$

$$\left(\frac{XY}{X^2+Y^2}\right)$$

$$\frac{x^2y^2}{x^4+y^4}$$