NONSTANDARD METHODS IN ALGEBRAIC GEOMETRY

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[Joint work with Lars Brünjes.]

The difficulty of many problems about algebraic varieties depends on the characteristic of the base field. Resolution of singularities (proved in characteristic zero, open in characteristic p) and Grothendieck's standard conjecture on the rationality of Künneth components (proved over finite fields, open in characteristic zero) are prominent examples. This is mostly due to the fact that some tools — like transcendental methods — are only available in characteristic zero while others — like Frobenius morphisms — only exist in characteristic p.

A link between the apparently so different worlds is provided by the ultraproduct

$$\prod_{p \in M, \mathcal{U}} \mathbb{F}_p$$

of the finite fields $\mathbb{F}_p$'s where M is an infinite set of primes. This is a field of characteristic zero which behaves in some sense like a finite field.

In order to use this ambiguity in algebraic geometry we started in a series of papers ([BS05], [BS07] and [BS08a/b]) a systematic investigation of how constructions in algebraic geometry behave under ultraproducts resp. enlargements.

In this talk we would like to give an overview about this work. We want to talk in more detail about algebraic cycles and the image of the cycles class map in etale cohomology and how we can use nonstandard methods there.

REFERENCES

[BS05]   Lars Brünjes and Christian Serpé. Enlargements of Categories. *Theory and Applications of Categories*, 14:357–398, 2005. http://www.tac.mta.ca/tac/

[BS07]   Lars Brünjes and Christian Serpé. Enlargements of schemes. *Log. Anal.*, 1(1):1–60, 2007.

[BS08a]   Lars Brünjes and Christian Serpé. *Etale and motivic cohomology and ultra products of schemes.* In preparation.

[BS08b]   Lars Brünjes and Christian Serpé. *Ultraproducts of cycles modulo homological equivalence.* In preparation.

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