ON HINDMAN SETS

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Let $(X,\cdot)$ be a groupoid. Let us call a set $H \subseteq X$ a *Hindman* set w.r.t. a sequence $(x_\alpha)_{\alpha<\kappa}$ of distinct elements of H if

$$((\cdots(x_{\alpha_{n+1}}x_{\alpha_n})\cdots x_{\alpha_2})x_{\alpha_1})x_{\alpha_0} \in H$$

whenever $\alpha_0 < \alpha_1 < \alpha_2 < \cdots < \alpha_n < \alpha_{n+1}$ and $n < \omega$. Hindman's theorem states that, if the operation is associative and right cancellative, each finite partition of X contains a piece which is a Hindman set w.r.t. some ω-sequence. This topic has close relationships with idempotent ultrafilters over X.

We study the existence of Hindman sets in infinite partitions and/or w.r.t. sequences of transfinite length. This leads to large cardinal properties like compactness. We consider also some situations with multiple structures and non-associative operations and conclude with some open questions.

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