# DYNAMICAL EQUIVALENCE ON $G^*$

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For every infinite group $G$, the remainder $G^* = \beta G \setminus G$ of the Stone-Čech compactification $\beta G$ of $G$ has a natural structure of $G$-space. The *orbit equivalence* ($(x, y) \in E \iff gx = y$ for some $g \in G$) determines the smallest by inclusion, closed in $G^* \times G^*$ equivalence $\check{E}$ on $G^*$ containing $E$ which is called a *dynamical equivalence*, and the factor-space $\gamma(G) = G^*/\check{E}$ is called a *corona* of $G$. To clarify the virtual equivalence $\check{E}$ we use the slowly oscillating functions (see [3]) and [4, Chapter 8]).

A function $f : G \longrightarrow [0, 1]$ is called *slowly oscillating* if, for any $g \in G$ and $\epsilon > 0$, there exists a finite subset $F$ of $G$ such that

$$| f(gx) - f(x) | < \epsilon$$

for every $x \in G \setminus F$. Then $(p, q) \in \check{E}$ if and only if, for every slowly oscilating function $f : G \longrightarrow [0, 1]$, $f^\beta(p) = f^\beta(q)$ where $f^\beta$ is the extension of $f$ to $\beta G$.

The space $\beta G$ has also a natural structure of compact right topological semigroup (see [1], [2]). Given $p \in G^*$, the orbit closure $\overline{Gp}$ is the left ideal $\beta Gp$ of $\beta G$. An ultrafilter $p \in G^*$ is called *strongly prime* if $p \notin \overline{G^* G^*}$.

**Theorem.** *Let $G$ be a countable discrete group, $p \in \beta G$ and $\check{p}$ be an $\check{E}$ -equivalence class containing $p$. Then*

(1) if $p$ is a $P$-point in $G^*$ then $\check{p} = \beta Gp$;

(2) if $p$ is strongly prime and $\check{p} = \beta Gp$ then $p$ is a $P$-point in $G$;

(3) there exist the strongly prime ultrafilters $p, q \in G^*$ such that $\check{p} = \check{q}$ but $\beta Gp \cap \beta Gq = \emptyset$;

(4) $\gamma(G)$ contains a topological copy of $\omega^* = \beta\omega \setminus \omega$;

(5) if $G$ is locally finite then $\gamma(G)$ contains a topological copy of $\omega^*$ which is a retract of $\gamma(G)$

(6) there exists a continuous surjective mapping $f : \gamma(G) \longrightarrow \gamma(\mathbb{N})$, where $\gamma(\mathbb{N}) = \{\check{p} \in \gamma(\mathbb{Z}) : \mathbb{N} \in p\}$.

## References

[1] N. Hindman and D. Strauss, Algebra in the Stone-Čech compactification - Theory and Applications, de Grueter, Berlin, 1998.
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[3] I. V. Protasov, Coronas of balleans, Topology Appl., 149 (2005), 149-160.
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