PRIME IDEAL THEOREM FOR WEAKLY DICOMPLEMENTED LATTICES

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A weakly dicomplemented lattice is a bounded lattice L equipped with two unary operations $^{\triangle}$ and $^{\triangledown}$ (called weak complementation and dual weak complementation) satisfying for all $x, y \in L$ the following equations:

(1) $x^{\triangle\triangle} \leq x$,	(1') $x^{\triangledown\triangledown} \geq x$,
(2) $x \leq y \implies x^{\triangle} \geq y^{\triangle}$,	(2') $x \leq y \implies x^{\triangledown} \geq y^{\triangledown}$,
(3) $(x \wedge y) \vee (x \wedge y^{\triangle}) = x$,	(3') $(x \vee y) \wedge (x \vee y^{\triangledown}) = x$.

A primary filter of L is a lattice filter F such for all $x \in L$ we have $x \in F$ or $x^{\triangle} \in F$. A primary ideal of L is a lattice ideal J of L such that for all $x \in L$ we have $x \in J$ or $x^{\triangledown} \in J$. We proved in [2] that if G is a filter and H an ideal such that $G \cap H = \emptyset$ then there is a primary filter $F \supseteq G$ and a primary ideal $J \supseteq H$ with $F \cap J = \emptyset$. The hope is to get an embedding of weakly dicomplemented lattices into concept algebras (defined below), and by then generate the equational theory of concept algebras. This is still an open problem.
A formal context is a triple (O, A, I) with $I \subseteq O \times A$. For $B \subseteq O$ and $C \subseteq A$ set

$$B' := \{a \in A \mid (o, a) \in I \ \forall o \in B\} \quad \text{and} \quad C' := \{o \in O \mid (o, a) \in I \ \forall a \in C\}.$$

A formal concept is a pair (B, C) with $B' = C$ and $C' = B$. The set $\mathfrak{B}(O, A, I)$ of all concepts of (O, A, I) forms a complete lattice (cf. [1]). A weak negation $^{\triangle}$ and a weak opposition $^{\triangledown}$ are defined on concepts by

$$(B, C)^{\triangle} := \big((O \setminus B)'', (O \setminus B)'\big) \quad \text{and} \quad (B, C)^{\triangledown} := \big((A \setminus C)', (A \setminus C)''\big).$$

$\big(\mathfrak{B}(O, A, I); \wedge, \vee, ^{\triangle}, ^{\triangledown}, 0, 1\big)$ is called the concept algebra of the context (O, A, I), where $\wedge$ and $\vee$ denote the meet and the join operations of the concept lattice. Concept algebras are genuine examples of weakly dicomplemented lattices ([3]). The main goal is to represent weakly dicomplemented lattices by concept algebras.

References

[1] B. Ganter & R. Wille. *Formal Concept Analysis. Mathematical Foundations.* Springer (1999).
[2] L. Kwuida. *Dicomplemented Lattices. A contextual generalization of Boolean algebras.* Dissertation, TU Dresden. Shaker Verlag (2004).
[3] R. Wille. *Boolean Concept Logic* in B. Ganter & G. Mineau (Eds.) *Conceptual Structures: Logical, linguistic and computational issues.* Proceedings ICCS 2000. LNAI **1867** Springer (2000) 317-331.

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