IDEMPOTENT ULTRAFILTERS AND FINER TOPOLOGIES ON $\beta\mathbb{N}$

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The so called strongly summable ultrafilters on $\mathbb{N}$ and their kin, the union ultrafilters on $[\omega]^{<\omega}$, are known for their many intriguing properties and their historic role in the development of the algebra in the Stone-Čech compactification. They are however practically the only examples of algebraically relevant ultrafilters the existence of which cannot be decided by the axioms of ZFC alone. In the spirit of Maryvonne Daguenet's "Propriété de Baire de βN muni d'une nouvelle topologie et application á la construction d'ultrafiltres" and Claude Laflamme's "Forcing with Filters and Complete Combinatorics" we introduce some new topological and algebraic tools for studying new kinds of idempotent ultrafilters; in particular we establish some independence results (from the axioms of ZFC) regarding the existence of such ultrafilters. These results are still work in progress begun while visiting Andreas Blass at the University of Michigan during the winter 2007/2008.

Related topics:
algebra in the Stone-Čech compactification, set theory and foundations, combinatorics.

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