ULTRAFILTERS, DETERMINACY, AND LARGE CARDINALS

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A two player game is said to be *determined* if one of the players has a winning strategy. The axiom of determinacy is the statement that every perfect information two player game of length ω on $\mathbb{N}$ is determined. Usually one studies restrictions of the axiom, obtained by limiting the complexity of the payoff sets, as the full axiom of determinacy contradicts the axiom of choice. Examples of standard complexities one considers include: Borel, analytic, projective, and within $L(\mathbb{R})$.

Ultrafilters are important in the study of determinacy for several reasons. First, it has been known for a long time that the full axiom of determinacy yields the existence of many natural ultrafilters: the filter on sets of reals generated by Turing cones is an ultrafilter (Martin), the filter on sets of countable ordinals generated by the club sets is an ultrafilter (Solovay), filters generated by clubs in $\mathcal{P}_\lambda(\kappa)$ are in many cases ultrafilters (Solovay, Becker, and others), and there are many other examples. Second, ultrafilters are used in studying models of determinacy. For example an analysis of the projective cardinals (Martin, Jackson) uses ultrafilters in an essential way. Third, proofs of determinacy for complexities beyond Borel require axioms positing the existence ultrafilters, a.k.a. large cardinals, in models of choice.

My talk will concentrate on the first and third reasons for the importance of ultrafilters in the study of determinacy. I will survey various constructions of ultrafilters under determinacy, proofs of determinacy from ultrafilters under choice, and more recent work involving ultrafilters under determinacy that are constructed using models of choice.

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