Proprietà delle potenze

a) $$\frac{(5^{-2}):\left(\frac{1}{5}\right)^{-1}\cdot 5^{2}}{\left(5^{0}-\frac{3}{5}\right)^{3}:\left(\frac{5}{2}\right)^{-4}:\left(\frac{4}{25}\right)^{-1}}+\frac{6^{3}:\left(\frac{1}{6}\right)^{-2}-5}{(2^{5})^{-3}\cdot\left[(2^{4})^{2}\right]^{2}}+\frac{\left(\frac{1}{2}\right)^{-4}}{\left(\frac{1}{2}\right)^{-6}}=\frac{5}{4}$$

b) $$\left(\frac{1}{3}\right)^{-\frac{1}{2}}\cdot 9^{-\frac{1}{3}}\cdot 27^{-\frac{1}{6}}=\frac{\sqrt[3]{3}}{3}$$

c) $$4^{-\frac{3}{2}}\cdot 8^{-\frac{1}{4}}\cdot\left(\frac{1}{16}\right)^{-\frac{1}{8}}=\frac{\sqrt[4]{8}}{16}$$

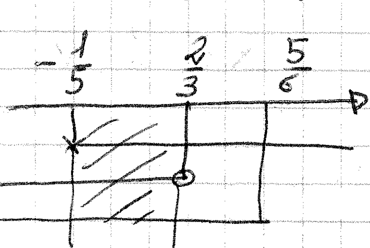
d) $$\frac{\sqrt{a}\cdot\sqrt[3]{a^{2}}\cdot\sqrt[4]{a^{3}}}{\sqrt[3]{a}\,\left(\sqrt[6]{a}\right)^{7}\cdot\sqrt[12]{a^{11}}}=\frac{\sqrt{a}}{a}\,,\qquad a>0$$

Equazioni e disequazioni di 1° grado intere e fratte

$\sqrt{5}(x-\sqrt{5})-\sqrt{2}(\sqrt{2}-x)=2\sqrt{10}$ $\qquad x=\sqrt{2}+\sqrt{5}$

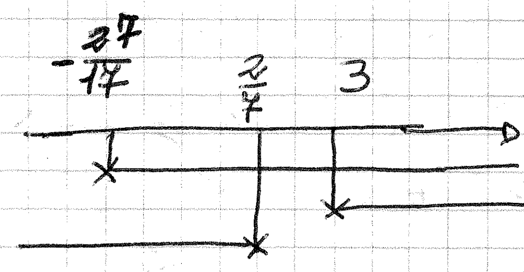
$$\frac{x+\sqrt{2}}{x-\sqrt{2}}-\frac{x-\sqrt{2}}{x+\sqrt{2}}=\frac{x\sqrt{2}+12}{x^{2}-2}\qquad x=2\sqrt{2}$$

$$\begin{cases}5x+1>0\\ 2-3x\geq 0\\ 6x-5<0\end{cases}$$



$-\frac{1}{5}<x\leq\frac{2}{3}$

$$\begin{cases}x+2-\dfrac{x-3}{3}+\dfrac{3x+1}{4}>1\\[2mm] \dfrac{x-1}{2}<\dfrac{3x-6}{3}\\[2mm] (2x-1)^{2}>(x+1)(4x-1)\end{cases}$$



$\emptyset$

$\frac{3x-4}{x+1}-2\leq 0$ $\quad ; \quad$ $\frac{1}{2}-\frac{x}{3}\geq\frac{3-x^{2}}{3x+1}$

$-1<x\leq 6$ $\qquad$ $x<-\frac{1}{3}\ \vee\ x\geq\frac{15}{7}$

Equazioni e disequazioni di 2° grado

a) $10^3 x(10^5+10^{-2}x) = 2x(10^{-2})^{-4}$; $x=0 \vee x=10^7$

b) $\left(\frac{1}{10^3}x\right)^2 - 3\cdot 10^{-2} = \frac{6}{10^2}$ $x=\pm 3\cdot 10^2$

c) $\left(5x+\frac{3}{4}\right)\left(5x-\frac{3}{4}\right)-\frac{27}{16}=0$ $x=\pm\frac{3}{10}$

c') $2x^2+1=0$, $\emptyset$

d) $\left(1-\frac{1}{3}\right)(1-x)-\frac{1}{2}(x+2)^2-\frac{2}{3}=10-3(x-2)^2$ $x=0 \vee x=\frac{88}{15}$

e) $(x-3)(1+3x)=6x^2+(1-3x)(2+3x)-1$; $x=-\frac{1}{2} \vee x=\frac{4}{3}$

f) $5x+x^2+3(2x^3+x^2+1)=6x^3$ $\emptyset$

g) $\frac{x+5}{3-x}-\frac{7-x}{2+x}+3=0$; $x=-\frac{1}{3} \vee x=7$

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h) $7x^2-x-6<0$; $-\frac{6}{7}<x<1$

i) $3x^2-2x-5>0$; $x<-1 \vee x>\frac{5}{3}$

l) $2x^2-5x+3\geq 0$; $x\leq 1 \vee x\geq\frac{3}{2}$

m) $(x-1)^3\geq 5+(x^2+5x-1)(x-2)$; $1\leq x\leq\frac{4}{3}$

n) $x^2-4x+13>0$, $\forall x\in\mathbb{R}$

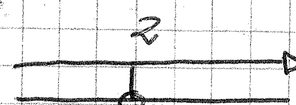
o) $29+(1+x)x\geq 11x$, $\forall x\in\mathbb{R}$

p) $x^2+x+8<0$, $\emptyset$

q) $3x^2-2x+2\leq 0$, $\emptyset$

r) $9x^2-30x+25>0$, $\forall x\neq\frac{5}{3}$

s) $4x^2+(x+2)^2\geq 4x(x+2)$, $\forall x\in\mathbb{R}$



t) $2x^2+5x\geq 0$ $x\leq-\frac{5}{2} \vee x\geq 0$; $3x^2-x<0$ $0<x<\frac{1}{3}$

2)

$x^2-3>0$; $x<-\sqrt{3} \vee x>\sqrt{3}$; $4x^2+5\geq 0$, $\forall x\in\mathbb{R}$

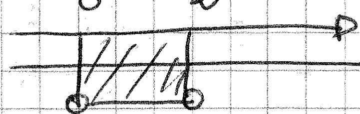
$1-8x^2\geq 0$; $-\frac{\sqrt{2}}{4}\leq x\leq \frac{\sqrt{2}}{4}$, $3x^2+1<0$, $\emptyset$

$2(2x+1)^2-3(x-2)^2\geq 20x$, $x\leq -\sqrt{2} \vee x\geq \sqrt{2}$

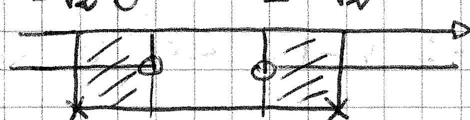
$\frac{x^2-1}{x^2-5x+6}\geq 1$; $\frac{7}{5}\leq x<2 \vee x>3$

$\frac{2x^2-1}{x^2-2x-3}\leq 2$; $x\leq -\frac{5}{4} \vee -1<x<3$

$\begin{cases} x^2-x+2>0 \\ x^2-2x\leq 0 \end{cases}$ $0\leq x\leq 2$

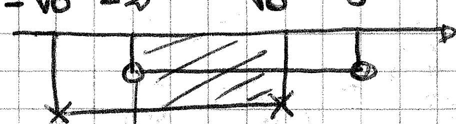


$\begin{cases} x^2-x\geq 0 \\ x^2-2<0 \end{cases}$



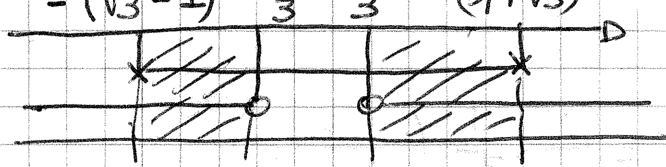
$-\sqrt{2}<x\leq 0 \vee$
$\vee\ 1\leq x<\sqrt{2}$

$\begin{cases} x^2-6x-16\leq 0 \\ x^2-6<0 \end{cases}$



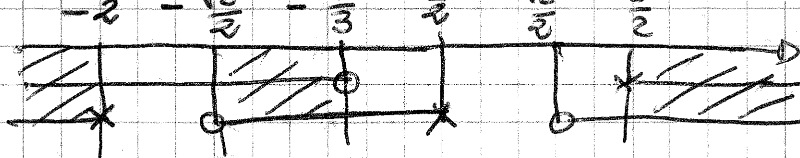
$-2\leq x<\sqrt{6}$

$\begin{cases} x^2-2x-2<0 \\ 9x^2-4\geq 0 \\ x^2+5>0 \end{cases}$



$-(\sqrt{3}-1)<x\leq -\frac{2}{3} \vee \frac{2}{3}\leq x<1+\sqrt{3}$

$\begin{cases} \frac{3x+1}{2x-3}\geq 0 \\ \frac{1-2x^2}{2x^2+3x-2}\leq 0 \end{cases}$



$x<-2 \vee -\frac{\sqrt{2}}{2}\leq x\leq -\frac{1}{3} \vee x>\frac{3}{2}$

Modulo o valore assoluto $|x|=\begin{cases} x, & \text{se } x\geq 0 \\ -x, & \text{se } x<0 \end{cases}$

Proprietà del valore assoluto

$\boxed{|a|\geq 0\ \forall a\in\mathbb{R}}$ $\boxed{\sqrt{a^2}=|a|\ \forall a\in\mathbb{R}}$; $\boxed{|a|=0 \Leftrightarrow a=0}$

$|a\cdot b|=|a|\cdot|b|$, $\forall a,b\in\mathbb{R}$ | $|a|=|-a|$, $\forall a\in\mathbb{R}$

$\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$ $\forall a\in\mathbb{R}$, $\forall b\neq 0$ | $\left|\frac{1}{b}\right|=\frac{1}{|b|}$, $\forall b\neq 0$

Verificare le seguenti uguaglianze dopo aver stabilito per quali valori della variabile hanno senso

- $$\left(\sqrt{\frac{x-1}{x+1}}-\sqrt{\frac{x+1}{x-1}}\right)^2=\frac{4}{x^2-1}$$

$$\begin{cases}\dfrac{x-1}{x+1}\geq 0\\[2mm] \dfrac{x+1}{x-1}\geq 0\end{cases};\quad \frac{x-1}{x+1}>0\ ;\quad x<-1 \lor x>1$$

- $$\sqrt{\frac{x^2-x}{x^2+2x+1}}:\sqrt{\frac{x^2-1}{x^2+x}}=\frac{|x|}{|x+1|}$$

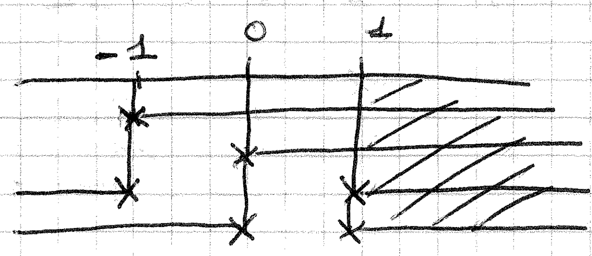
$$\begin{cases}\dfrac{x^2-x}{(x+1)^2}\geq 0\\[2mm] \dfrac{x^2-1}{x^2+x}>0\end{cases};\quad x<-1 \lor -1<x<0 \lor x>1$$

- $$\sqrt[4]{\left(\frac{y^4}{4}+y^2+1\right)\cdot\left(\frac{y^4-4}{2\sqrt{2}}\right)^2\cdot\frac{2}{(y^3-2y)^2}}=\frac{y^2+2}{2|y|}\sqrt{|y|}$$

$y\neq 0$
$y\neq\sqrt{2}$
$y\neq-\sqrt{2}$

- $$\frac{\dfrac{(x+1)^x}{x^{(x+1)}}}{\left(1-\dfrac{1}{x^2}\right)^{(x-1)}}=\frac{x^2-1}{x^3}\left(\frac{x}{x-1}\right)^x$$

$$\begin{cases}x+1>0\\ x>0\\ 1-\dfrac{1}{x^2}>0\\[2mm] \dfrac{x}{x-1}>0\end{cases}$$



$x>1$