Analisi Matematica II - Corso di Laurea in Fisica

Esercizi sugli integrali.

Integrali risolubili in maniera elementare o con cambiamento di variabile.

1) $\displaystyle\int \frac{e^x + e^{-x}}{1+(e^x - e^{-x})^2}\ dx$; **2)** $\displaystyle\int \frac{1}{\sqrt{2x} - \sqrt{x}}\ dx$; **3)** $\displaystyle\int \frac{1}{\sqrt{x+1} - \sqrt{x}}\ dx$

4) $\displaystyle\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\ dx$; **5)** $\displaystyle\int e^{(x+1)^2}\, e^{x^2+2x}\,(x+1)\ dx$; **6)** $\displaystyle\int \left(\frac{1+2\sqrt{x}}{\sqrt{x}}\right) e^{\sqrt{x}+x}\ dx$

7) $\displaystyle\int \frac{\cos x\ \sin^2 x}{1+\sin^2 x}\ dx$; **8)** $\displaystyle\int \sqrt{\sin^4 x - \cos^2 x}\ \sin 2x\ dx$; **9)** $\displaystyle\int \frac{x}{1+(1+x)^2}\ dx$

10) $\displaystyle\int e^{x+e^x}\ dx$;

Calcolare il valore dei seguenti integrali definiti:

11) $\displaystyle\int_0^{4\pi} [\sin x]\ dx$; **12)** $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\ dx$, dove $f(x) = \begin{cases} \dfrac{x}{|x|}\, e^{\sin^2 x}\ \sin 2x, & \text{se } x \neq 0 \\ 0, & \text{se } x = 0; \end{cases}$;

13) $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 x\ dx$ **14)** $\displaystyle\int_{-\pi}^{\pi} (|\sin x| + \cos x)(|\cos x| + \cos x)\ dx$;

15) $\displaystyle\int_{-2}^{2} |x^3 - x|\, dx$; **16)** $\displaystyle\int_{-\pi}^{\pi} \sin^3 x\ dx$;

17) $\displaystyle\int_0^{\frac{3\pi}{2}} e^{|\sin x|}\ \cos x\ dx.$

Risultati

1) $\arctan(e^x - e^{-x}) + C$; **2)** $\frac{2}{\sqrt{2}-1}\sqrt{x} + C$; **3)** $\frac{2}{3}\left[\sqrt{(x+1)^3} + \sqrt{x^3}\right] + C$; **4)** $2\arctan\sqrt{x} + C$;

5) $\frac{1}{4}e^{2x^2+4x+1} + C$; **6)** $2e^{\sqrt{x}+x} + C$; **7)** $\sin x - \arctan(\sin x) + C$; **8)** $\frac{1}{3}\sqrt{(\sin^4 x - \cos^4 x)^3}$;

9) $\frac{1}{2}\log[1+(x+1)^2] - \arctan(x+1) + C$; **10)** $e^{e^x} + C$; **11)** $-\pi$; **12)** $2(e-1)$;

13) $1 - \frac{\pi}{4}$; **14)** 2; **15)** 5; **16)** 0; **17)** $1-e$.

Calcolare i seguenti integrali.

1) $\displaystyle\int e^{2x}\,\sin x\ dx;$ **2)** $\displaystyle\int \sqrt{x}\,e^{\sqrt{x}}\ dx;$ **3)** $\displaystyle\int \log(1+\sqrt{x})\ dx$

4) $\displaystyle\int e^{x}\ \cos^2 x\ dx;$ **5)** $\displaystyle\int \frac{1}{x(a+bx^n)}\ dx;$ **6)** $\displaystyle\int x^3\sqrt{x^2\,+\,a^2}\,dx$

7) $\displaystyle\int \sin^4 x\ dx;$ **8)** $\displaystyle\int \frac{x^2+1}{1+\sqrt{x-1}}\ dx;$ **9)** $\displaystyle\int \cos(kx)\,\cos(mx)\,dx,\ (k,\,m\in\mathbb{Z});$

10) $\displaystyle\int \cos(kx)\,\sin(mx)\,dx,\ (k,\,m\in\mathbb{Z});$ **11)** $\displaystyle\int x\,\cos kx\,dx,\ (k\in\mathbb{Z});$ **12)** $\displaystyle\int \frac{\sin\left[x(n\,+\,\frac{1}{2})\right]}{\sin\frac{x}{2}}\,dx,\ (n\in\mathbb{N}).$

Risultati e suggerimenti.

1) $\dfrac{1}{5}\,e^{2x}\,(2\,\sin x\,-\,\cos x)\,+\,C;$

2) $2\,e^{\sqrt{x}}(x\,-\,2\,\sqrt{x}\,+\,2)\,+\,C;$

3) $(x-1)\log(1+\sqrt{x})\,-\,\dfrac{1}{2}\,x\,+\,\sqrt{x}\,+\,C;$

4) $e^{x}\left[\cos^2 x\,+\,\dfrac{\sin 2x}{5}\,-\,\dfrac{2}{5}\,\cos 2x\right]\,+\,C;$

5) (porre $x^n=t$), $\dfrac{1}{an}\,\log\dfrac{x^n}{a\,+\,bx^n}\,+\,C;$

6) (porre $x^2=t$), $\dfrac{2}{15}\,\sqrt{(x^2-a^2)^5}\,+\,\dfrac{x^2}{3}\sqrt{(x^2-a^2)^3};$

7) $-\sin^3 x\,\cos x\,+\,\dfrac{3}{8}\,x\,-\,\dfrac{3\,\sin 4x}{32}\,+\,C;$

8) $\dfrac{1}{4}\,(x-1)^2\,+\,\dfrac{1}{3}\,(x-1)^{\frac{3}{2}}\,+\,\dfrac{3}{2}\,(x-1)\,-\,3\,\sqrt{x-1}\,+\,5\,\log(1\,+\,\sqrt{x-1})\,+\,C;$

9) 0 se $m\neq 0$, π se $m=k\neq 0$;

10) 0 se $m\neq k$, π se $m=k\neq 0$;

11) 0 se k pari diverso da zero, $-\dfrac{2}{k^2}$ se k dispari;

12) π, si utilizzi l'identità, che si dimostra per induzione, $\displaystyle\frac{1}{2}\,+\,\sum_{h=1}^{n}\cos h\alpha\,=\,\frac{\sin(n+\frac{1}{2})\alpha}{2\,\sin(\frac{\alpha}{2})}.$