1.

CE., SGN, ZERI

$$\begin{array}{ccccccccc} - & \times & - & 0 & + & 0 & + \\ & -1 & & 0 & & 1 & \end{array}$$

LIM per $x \to -1$ $f(x) \to -\infty$ asintoto verticale

per $x \to \pm\infty$ $f(x) \sim x \to \pm\infty$

ricerca asintoti obliqui

per $x \to +\infty$ $f(x) - x = x\left(\sqrt{\frac{x-1}{x+1}} - 1\right) =$

$$= x \frac{\sqrt{x-1} - \sqrt{x+1}}{\sqrt{x+1}} = \frac{-2x}{\sqrt{x+1}\left(\sqrt{x-1} + \sqrt{x+1}\right)}$$

$$\sim \frac{-2x}{\sqrt{x} \cdot 2\sqrt{x}} \to -1 \qquad y = x - 1 \text{ asintoto}$$

per $x \to -\infty$ $f(x) - x = x\left(\sqrt{\frac{1-x}{-1-x}} - 1\right) =$

$$= x \frac{\sqrt{1-x} - \sqrt{-1-x}}{\sqrt{-1-x}} = \frac{2x}{\sqrt{1-x}\left(\sqrt{1-x} + \sqrt{-1-x}\right)}$$

$$\sim \frac{2x}{\sqrt{x} \cdot 2\sqrt{-x}} \to -1 \qquad y = x + 1 \text{ asintoto}$$

DRV

$$f'(x) = \sqrt{\left|\frac{x-1}{x+1}\right|} + \frac{x}{2}\sqrt{\left|\frac{x+1}{x-1}\right|}\, \mathrm{sgn}\left(\frac{x-1}{x+1}\right) \frac{x+1-x+1}{(x+1)^2} =$$

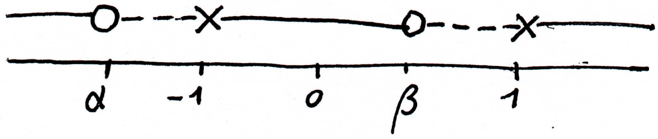
$$= \sqrt{\left|\frac{x-1}{x+1}\right|} + \frac{x}{(x+1)^2}\sqrt{\left|\frac{x+1}{x-1}\right|}\, \mathrm{sgn}\left(\frac{x-1}{x+1}\right) =$$

$$= \sqrt{\left|\frac{x-1}{x+1}\right|}\left(1 + \frac{x}{(x+1)^2}\left|\frac{x+1}{x-1}\right| \mathrm{sgn}\left(\frac{x-1}{x+1}\right)\right) =$$

$$= \sqrt{\left|\frac{x-1}{x+1}\right|}\left(1 + \frac{x}{x^2-1}\right) = \sqrt{\left|\frac{x-1}{x+1}\right|}\, \frac{x^2+x-1}{x^2-1}$$

$$x \neq \pm 1$$

SGN $f'$

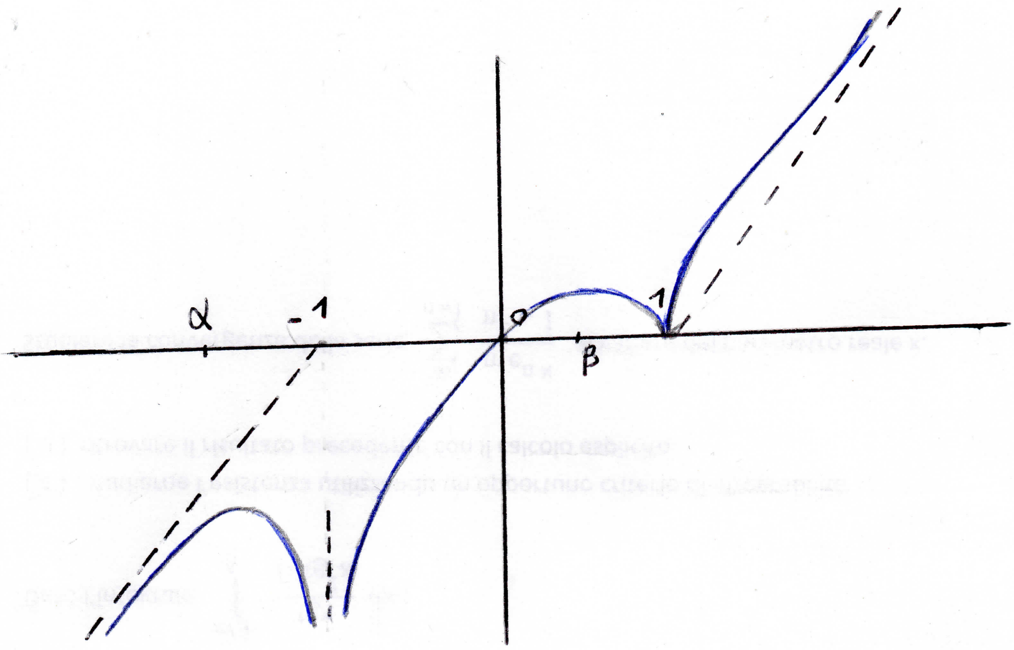



$$\alpha = \frac{-1-\sqrt{5}}{2}$$

$$\beta = \frac{-1+\sqrt{5}}{2}$$

per $x \to 1$ $\quad f'(x) \sim \dfrac{\sqrt{|x-1|}}{2\sqrt{2}\,(x-1)} = \dfrac{1}{2\sqrt{2}} \quad \dfrac{\text{sgn}(x-1)}{\sqrt{|x-1|}}$

pto di cuspide




2.

per parti:

$$-\frac{\lg(x^2-2x+2)}{x} + \int \frac{2x-2}{x(x^2-2x+2)}\,dx$$

$$\frac{2x-2}{x(x^2-2x+2)} = \frac{A}{x} + \frac{Bx+C}{x^2-2x+2} = \dots = -\frac{1}{x} + \frac{x}{(x-1)^2+1}$$

$$y = -\frac{\lg(x^2-2x+2)}{x} - \lg|x| + \frac{1}{2}\int \frac{2x-2+2}{(x-1)^2+1}\,dx$$

$$= -\frac{\lg(x^2-2x+2)}{x} - \lg|x| + \frac{1}{2}\lg(x^2-2x+2) + \text{arctg}(x-1) + c$$

3.

$$\operatorname{arctg} x = x - \frac{1}{3}x^3 + o(x^4) \qquad \lg(1+x) = x - \frac{1}{2}x^2 + o(x^2)$$

$$\lg(1 + x \operatorname{arctg} x) = \lg\left(1 + x^2 - \frac{1}{3}x^4 + o(x^4)\right) =$$

$$= \left(x^2 - \frac{1}{3}x^4\right) - \frac{1}{2}x^4 + o(x^4) = x^2 - \frac{5}{6}x^4 + o(x^4)$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2) \rightarrow e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^4)$$

Numeratore $\sim -\frac{4}{3}x^4$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2) \qquad \cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)$$

$$\sqrt{1-2x^2} = 1 - x^2 - \frac{1}{2}x^4 + o(x^4) \qquad \cos^2 x = 1 - x^2 + \frac{1}{3}x^4 + o(x^4)$$

Denominatore $\sim -\frac{5}{6}x^4$

Limite $= \frac{8}{5}$

4. $\sqrt{65} = \sqrt{64+1} = 8\sqrt{1 + 1/64}$

Formula di Taylor per $\sqrt{1+x}$, con $x_0 = 0$, $n = 2$, $x = 1/64$

Calcoliamo le derivate successive di $\sqrt{1+x} = (1+x)^{1/2}$

$$D = \frac{1}{2}(1+x)^{-1/2} \qquad D^2 = -\frac{1}{4}(1+x)^{-3/2} \qquad D^3 = \frac{3}{8}(1+x)^{-5/2}$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{x^3}{16(1+\xi)^{5/2}}$$

$$\sqrt{1 + 1/64} = 1{,}00778198\ldots + E \qquad E = \frac{1}{16 \cdot 64^3 (1+\xi)^{5/2}}$$

$$0 < \xi < 1/64$$

errore positivo $\rightarrow$ appross. per difetto

$$E < \frac{1}{16 \cdot 64^3} < 3 \cdot 10^{-7} \qquad \left(\text{perché } 1+\xi > 1 \rightarrow (1+\xi)^{5/2} > 1 \rightarrow \frac{1}{(1+\xi)^{5/2}} < 1\right)$$

$$\sqrt{65} \cong 8{,}0622558\ldots$$

$$8E < 24 \cdot 10^{-7} = 0{,}0000024$$

l'errore non influisce sulle prime 5 cifre decimali.