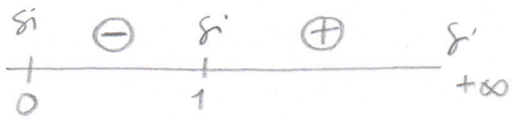
Soluzioni

1. $f(t) = \dfrac{\lg t}{\sqrt{|t^2-1|}}$



per $t \to 0$ $\quad f(t) \sim \lg t \;;\; |\lg t| < \dfrac{1}{t^\alpha}$ $\quad$ Scegliamo $\alpha < \frac{1}{2}$

per $t \to 1$ $\quad f(t) \sim \dfrac{t-1}{\sqrt{2|t-1|}} \to 0$ $\quad$ Punto disc. eliminabile

per $t \to +\infty$ $\quad f(t) \sim \dfrac{\lg t}{t} > \dfrac{1}{t}$. Non integrabile

$F(x) = \int_1^x f(t)\,dt$

C.E. $\quad x \geq 0$

SGN $\quad F(x)$ positiva , $F(1) = 0$

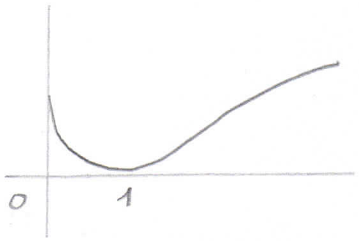
LIM $\quad$ per $x \to +\infty$ $\quad F(x) \to +\infty$

DRV $\quad F'(x) = \dfrac{\lg x}{\sqrt{|x^2-1|}}$

per $x \to 1$ $\quad F'(x) \to 0$

per $x \to 0$ $\quad F'(x) \to -\infty$

per $x \to +\infty$ $\quad F(x) \to 0$ $\quad$ NO ASINT.



2. Si pone $\sqrt{1+x} = t \;\to\; x = t^2 - 1 \,,\; dx = 2t\,dt$

$$y = \int \frac{2t(t^2-1)}{(t+1)^2}\,dt = \int \frac{2t(t-1)}{t+1}\,dt = 2\int (t-2)\,dt + 4\int \frac{dt}{t+1} =$$

$$= t^2 - 4t + 4\lg|t+1| + c = 1 + x - 4\sqrt{1+x} + 4\lg\left(\sqrt{1+x}+1\right) + c$$

3. $\operatorname{sen} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!}$ $\quad$ Criterio rapporto : $\left|\dfrac{a_{n+1}}{a_n}\right| = \dfrac{x^2}{(2n+3)(2n+2)} \to 0 < 1$

La serie converge $\forall x \in \mathbb{R}$

$\dfrac{\operatorname{sen} x}{x} = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n+1)!}$ $\qquad I = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n+1)!\,(2n+1)}$ $\quad$ Vale teor. Leibniz

$|I - S_n| < \dfrac{1}{(2n+3)!\,(2n+3)} < \dfrac{1}{10^3}$ $\quad$ per $n \geq 2$.

$I \sim 1 - \dfrac{1}{3!\,3} + \dfrac{1}{5!\,5} = \cdots$

Soluzioni [2]

1. $f(t) = \dfrac{\lg t}{t^2-1}$

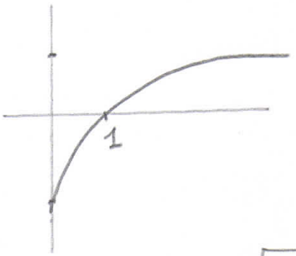
$$\begin{array}{ccccc} \text{si} & \oplus & \text{si} & \oplus & \text{si} \\ 0 & & 1 & & +\infty \end{array}$$

per $t \to 0$ $\quad f(t) \sim -\lg t < \dfrac{1}{t^\alpha}$; scegliamo $\alpha < 1$

per $t \to 1$ $\quad f(t) \sim \dfrac{t-1}{(t-1)(t+1)} \to \dfrac{1}{2}$; punto disc. elimin.

per $t \to +\infty$ $\quad f(t) \sim \dfrac{\lg t}{t^2} < \dfrac{t^\alpha}{t^2} = \left(\dfrac{1}{t}\right)^{2-\alpha}$; si sceglie $\alpha > 0$ in modo che sia $2-\alpha > 1$, cioè $\alpha < 1$.

$$F(x) = \int_1^x f(t)\,dt$$



C.E. $\quad x \geq 0$

SGN $\quad$ negativa in $[0,1)$, positiva in $(1,+\infty)$; $F(1) = 0$

LIM $\quad$ per $x \to +\infty$ $\quad F(x) \to \int_1^{+\infty} f(t)\,dt \in \mathbb{R}^+$

DRV $\quad F'(x) = \dfrac{\lg x}{x^2-1}$ $\quad$ per $x \to 1$ $\quad F'(x) \to \dfrac{1}{2}$

per $x \to 0$ $\quad F'(x) \to +\infty$

2. Si pone $\sqrt{1-x} = t \to x = 1-t^2, \quad dx = -2t\,dt$

$$y = \int \frac{2t(t^2-1)}{(t+1)^2}\,dt$$

COME IN [1]
VEDERE SOLUZIONE PRECEDENTE

3. $\lg(1+x) = \displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

Criterio rapporto $\left|\dfrac{a_{n+1}}{a_n}\right| = |x| \dfrac{n}{n+1} \to |x|$

La serie converge in $(-1,1)$.
Per $x = 1$ converge per Leibniz.
Per $x = -1$, $\displaystyle\sum_{n=1}^{\infty} -\frac{1}{n}$ diverge.

$$\frac{\lg(1+x)}{x} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n-1}}{n}$$

$$I = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}.$$ Vale il teor. di Leibniz

$$|I - S_n| < \frac{1}{(n+1)^2} < \frac{1}{10} \quad \text{per } n \geq 3$$

$$I \sim 1 - \frac{1}{4} + \frac{1}{9} = \dots$$