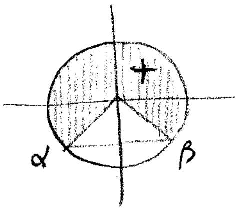
Soluzioni [1]

1. $f(x) = -2\,\mathrm{sen}^2 x + 2\,\mathrm{sen}\, x + 1, \quad x \in [0, 2\pi]$

$f(x) \geq 0$ per $\frac{1-\sqrt{3}}{2} \leq \mathrm{sen}\, x \leq 1$

$f'(x) = 2\cos x\,(1 - 2\,\mathrm{sen}\, x)$

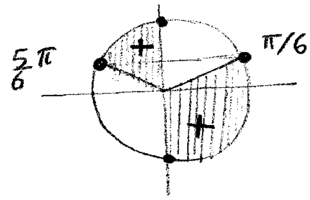
sgn $f(x)$

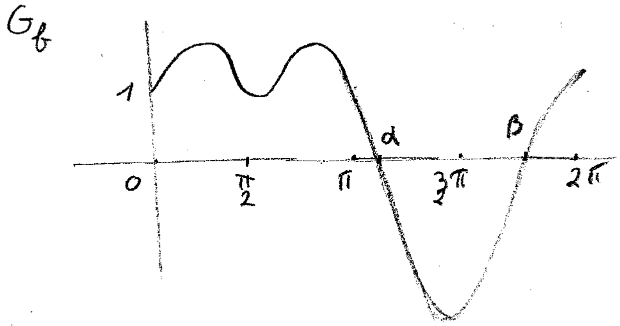


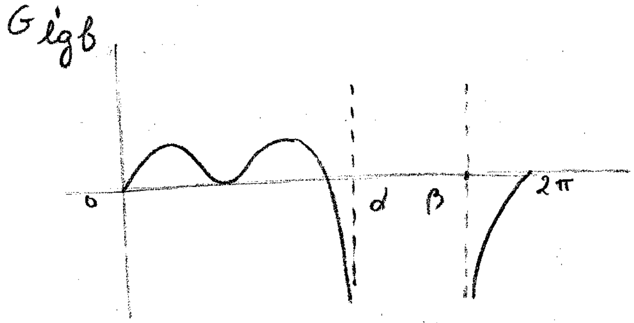
$\alpha = \pi - \mathrm{arcsen}\,\frac{1-\sqrt{3}}{2}$

$\beta = 2\pi + \mathrm{arcsen}\,\frac{1-\sqrt{3}}{2}$

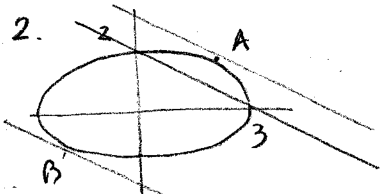
sgn $f'(x)$







2.



retta coeff. ang. $-2/3$

$y = \pm \frac{2}{3}\sqrt{9-x^2}\,, \quad y' = \mp \frac{2}{3}\frac{x}{\sqrt{9-x^2}} = -\frac{2}{3}$

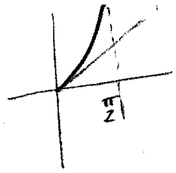
$A = \left(\frac{3}{\sqrt{2}}, \sqrt{2}\right) \qquad y = \sqrt{2} - \frac{2}{3}\left(x - \frac{3}{\sqrt{2}}\right)$

$B = \left(-\frac{3}{\sqrt{2}}, -\sqrt{2}\right) \qquad y = -\sqrt{2} - \frac{2}{3}\left(x + \frac{3}{\sqrt{2}}\right)$.

3. $I = \frac{\pi}{12}\frac{\mathrm{sen}\,\xi}{\xi}\,, \quad \xi \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$f(x) = \frac{\mathrm{sen}\,x}{x}\,, \quad x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$f'(x) = \frac{x\cos x - \cos x}{x^2} \geq 0 \iff \mathrm{tg}\, x \leq x$ falsa



Dunque $\frac{\mathrm{sen}\,x}{x}$ è decrescente; $\frac{3\sqrt{3}}{2\pi} < \frac{\mathrm{sen}\,x}{x} < \frac{4}{\sqrt{2}\,\pi}$

$\sqrt{2}/8 < I < \sqrt{2}/6$

4. $\int \mathrm{arctg}\,\frac{t-1}{t}\,dt = t\,\mathrm{arctg}\,\frac{t-1}{t} - \int \frac{t}{2t^2-2t+1}\,dt = t\,\mathrm{arctg}\,\frac{t-1}{t} - \frac{1}{4}\int \frac{4t-2+2}{2t^2-2t+1}\,dt$

$= t\,\mathrm{arctg}\,\frac{t-1}{t} - \frac{1}{4}\lg(2t^2-2t+1) - \int \frac{dt}{(2t-1)^2+1} =$

$= t\,\mathrm{arctg}\,\frac{t-1}{t} - \frac{1}{4}\lg(2t^2-2t+1) - \frac{1}{2}\,\mathrm{arctg}(2t-1) + c$

$F(x) = x\,\mathrm{arctg}\,\frac{x-1}{x} - \frac{1}{4}\lg(2x^2-2x+1) - \frac{1}{2}\,\mathrm{arctg}(2x-1) - \frac{\pi}{8}$

$F'(x) = \mathrm{arctg}\,\frac{x-1}{x} > 0$. Inoltre $\int_1^{+\infty} \mathrm{arctg}\,\frac{t-1}{t}\,dt = +\infty$ perchè $f(t) \not\to 0$.

min F = inf F = $F(1) = 0$

max F non $\exists$

sup $F = +\infty$

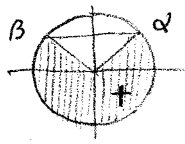
Soluzioni [2]

1. $f(x) = -2\,\text{sen}^2 x - 2\,\text{sen}\, x + 1, \quad x \in [0, 2\pi]$

$f(x) \geq 0$ per $-1 \leq \text{sen}\, x \leq \dfrac{-1+\sqrt{3}}{2}$.

$f'(x) = -2\cos x\,(2\,\text{sen}\, x + 1)$

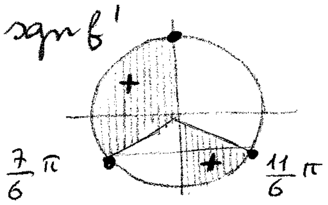
sgn f

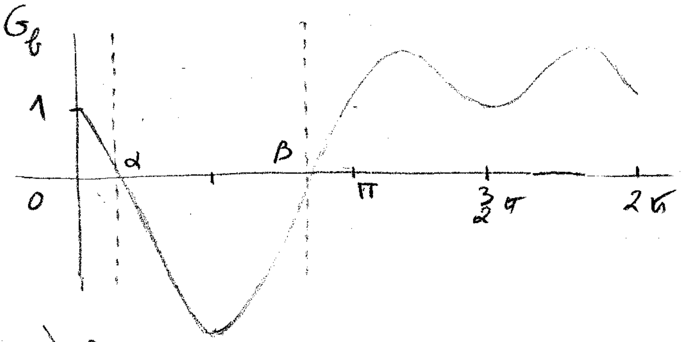


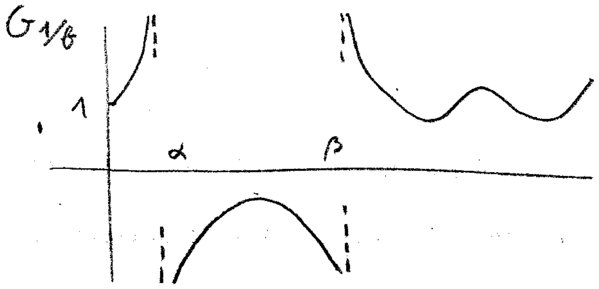
$\alpha = \text{arcsen}\, \dfrac{\sqrt{3}-1}{2}$

$\beta = \pi - \text{arcsen}\, \dfrac{\sqrt{3}-1}{2}$

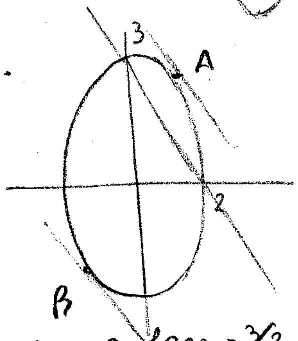
sgn f'







2.



c. angolare $-3/2$

$y = \pm \dfrac{3}{2}\sqrt{4-x^2}$, $\quad y' = \mp \dfrac{3}{2} \dfrac{x}{\sqrt{4-x^2}} = -\dfrac{3}{2}$

$A = \left(\sqrt{2}, \dfrac{3}{\sqrt{2}}\right) \qquad y = \dfrac{3}{\sqrt{2}} - \dfrac{3}{2}(x - \sqrt{2})$

$B = \left(-\sqrt{2}, -\dfrac{3}{\sqrt{2}}\right) \qquad y = -\dfrac{3}{\sqrt{2}} - \dfrac{3}{2}(x + \sqrt{2})$

3. $I = \dfrac{\pi}{6} \dfrac{\text{sen}\,\xi}{\xi}, \quad \xi \in \left(\dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$

nell'intervallo la fz. $\dfrac{\text{sen} x}{x}$ è decrescente (V. soluzioni [1]), dunque $\dfrac{3\sqrt{3}}{2\pi} < \dfrac{\text{sen} x}{x} < \dfrac{3}{\pi}$

In conclusione, $\dfrac{\sqrt{3}}{4} < I < \dfrac{1}{2}$.

4. $\displaystyle\int \text{arctg}\, \frac{t}{t+1}\, dt = t\, \text{arctg}\, \frac{t}{t+1} - \int \frac{t}{2t^2+2t+1}\, dt = t\, \text{arctg}\, \frac{t}{t+1} - \frac{1}{4} \int \frac{4t+2-2}{2t^2+2t+1}\, dt$

$\displaystyle = t\, \text{arctg}\, \frac{t}{t+1} - \frac{1}{4} \lg(2t^2+2t+1) + \int \frac{dt}{(2t+1)^2+1}$

$\displaystyle = t\, \text{arctg}\, \frac{t}{t+1} - \frac{1}{4} \lg(2t^2+2t+1) + \frac{1}{2} \text{arctg}(2t+1) + c$

$\displaystyle F(x) = x\, \text{arctg}\, \frac{x}{x+1} - \frac{1}{4} \lg(2x^2+2x+1) + \frac{1}{2} \text{arctg}(2x+1) - \frac{\pi}{8}$

$\displaystyle F'(x) = \text{arctg}\, \frac{x}{x+1} > 0$. Inoltre $\displaystyle\int_0^\infty \text{arctg}\, \frac{t}{t+1}\, dt = +\infty$ perchè $f(t) \not\to 0$.

$\min F = \inf F = F(0) = 0$

$\max F$ non $\exists$

$\sup F = +\infty$.