Soluzioni

1\. C. E. $(-\infty,-1] \cup [1,+\infty)$

La fz è dispari, quindi possiamo limitarci a studiarla per le x positive.
I calcoli successivi sono dunque fatti per $x \in [1,+\infty)$.

SGN $f(x) \geq 0 \iff x \geq \sqrt{2}$

LIM per $x \to +\infty$ $f(x) \sim x^2 \to +\infty$ senza asintoto ; $f(1) = -1$.

DRV $f'(x) = \dfrac{2x^2-1-\sqrt{x^2-1}}{\sqrt{x^2-1}}$, $(x \neq 1)$

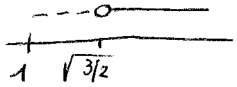
$x = 1$ punto a tg verticale

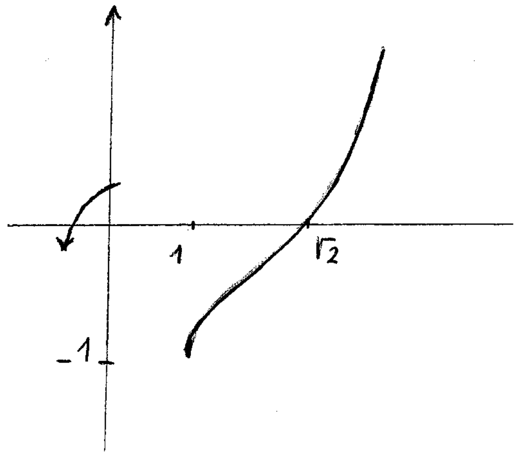
$f'(x) \geq 0 \iff \sqrt{x^2-1} \leq 2x^2-1 \iff$

$$\begin{cases} x \geq 1 \\ 2x^2-1 \geq 0 \\ 4x^4-5x^2+2 \geq 0 \;(*) \end{cases} \iff x \geq 1$$

(*) posto $t = x^2$:
$4t^2-5t+2 > 0 \quad \forall t$
perché $\Delta < 0$

DRV² $f''(x) = \dfrac{x(2x^2-3)}{(x^2-1)^{3/2}}$





2\. $x^2+2x+2 > 0 \quad \forall x \in \mathbb{R} \; (\Delta < 0)$; la funzione è continua e dunque l'integrale esiste in senso proprio.

$\sqrt{x^2+2x+2} = x + t$

$x = \dfrac{t^2-2}{2(1-t)}$, $dx = -\dfrac{t^2-2t+2}{2(1-t)^2}\,dt$, $\sqrt{x^2+2x+2} = -\dfrac{t^2-2t+2}{2(1-t)}$

$$y = -\int_{\sqrt{2}}^{\sqrt{2}+2} \frac{t^2-2}{2(1-t)^2}\,dt = -\int_{\sqrt{2}}^{\sqrt{2}+2} \left(1 + \frac{2t-2}{t^2-2t+1} - \frac{1}{t^2-2t+1}\right) dt =$$

$$= -\left[t + \lg(t-1)^2 + \frac{1}{t-1}\right]_{\sqrt{2}}^{\sqrt{2}+2} = 2\lg\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

3. Con le notazioni viste a lezione: $A(x) = \operatorname{sen} x$, $e^{A(x)} = e^{\operatorname{sen} x}$.

$$(y e^{\operatorname{sen} x})' = -\cos x \operatorname{sen} x \, e^{\operatorname{sen} x}$$

$$\int \cos x \operatorname{sen} x \, e^{\operatorname{sen} x} dx \underset{\substack{\operatorname{sen} x = t \\ \cos x \, dx = dt}}{=} \int t e^t dt \underset{\text{per parti}}{=} t e^t - \int e^t dt = t e^t - e^t + c =$$

$$= e^{\operatorname{sen} x} (\operatorname{sen} x - 1) + c$$

$$y e^{\operatorname{sen} x} = e^{\operatorname{sen} x} (\operatorname{sen} x - 1) + c \Rightarrow y = \operatorname{sen} x - 1 + c e^{-\operatorname{sen} x}.$$

4. $\left| \frac{a_{n+1}}{a_n} \right| = |x| \frac{\lg n}{\lg(n+1)} \to |x|$

La serie converge (ass.) per $x \in (-1, 1)$.

Per $x = 1$, $\sum_{n=2}^{\infty} \frac{(-1)^n}{\lg n}$ converge per il teorema di Leibniz.

Per $x = -1$, $\sum_{n=2}^{\infty} \frac{1}{\lg n}$ diverge $\left(\frac{1}{\lg n} > \frac{1}{n} \right)$.