

## soluzioni [ A ]

1.  $\lim_{x \rightarrow x_0} \varphi(x) = L$  ,  $\lim_{t \rightarrow L} f(t) = M$

$\varphi(x) \neq L$  in  $U(x_0) - \{x_0\}$  oppure  $f(t)$  continua in  $L$

$\Rightarrow \lim_{x \rightarrow x_0} f(\varphi(x)) = M$

2.  $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 0$

3.  $1/2$

4.  $\frac{\operatorname{arctg} 2x}{4} + \frac{x}{2(4x^2 + 1)} + c$  .

5.  $(1+i)/\sqrt[3]{2}$

6.  $2(x - \pi/4)$

## soluzioni [ B ]

1.  $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$  ,  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

2. (i)  $\forall x \in A$  ,  $f(x) \leq L$  (ii)  $\forall \varepsilon > 0$  ,  $\exists \bar{x} \in A : f(\bar{x}) > L - \varepsilon$

3.  $1/2$

4.  $\frac{\operatorname{arctg} x/2}{16} + \frac{x}{8(x^2 + 4)} + c$  .

5.  $1 \pm i\sqrt{3}$  ,  $-2$  ,  $0$

6.  $(1-x)/3$

soluzioni [ C ]

1.  $\sqrt[n]{|a_n|} \rightarrow \begin{cases} L < 1 \\ L > 1 \\ 1 \end{cases} \Rightarrow \sum_{n=1}^{\infty} a_n \begin{cases} \text{converge assolutamente} \\ \text{non converge} \\ \text{non si può decidere} \end{cases}$
2.  $A, B \subset \mathbb{Q}$  (i)  $\forall a \in A, \forall b \in B \quad a \leq b$  (ii)  $\forall \varepsilon > 0, \exists \bar{a} \in A, \exists \bar{b} \in B \quad \bar{b} - \bar{a} < \varepsilon$
3.  $1/4$
4.  $\frac{\operatorname{arctg} \sqrt{2} x}{2\sqrt{2}} + \frac{x}{2(2x^2+1)} + c .$
5.  $(1+i)/\sqrt[3]{2}$
6.  $x - 1$

soluzioni [ D ]

1.  $f(x), x \in (0, +\infty)$  funzione continua, decrescente, infinitesima all'infinito.  
 $\int_1^{\infty} f(x) dx$  e  $\sum_{n=1}^{\infty} f(n)$  hanno la stessa natura ( entrambi convergono o entrambi divergono )
2. (i)  $\forall x \in A, f(x) \geq 1$  (ii)  $\forall \varepsilon > 0, \exists \bar{x} \in A : f(\bar{x}) < 1 + \varepsilon$
3.  $1/4$
4.  $\frac{\sqrt{2} \operatorname{arctg} x / \sqrt{2}}{8} + \frac{x}{4(x^2+2)} + c .$
5.  $1 \pm i\sqrt{3}, -2, 0$
6.  $2(x - 1/2) / \sqrt{3}$