Soluzioni [1]

1. C.E. $\begin{cases} x>0 \\ -1\le \lg_{1/2} x \le 1 \\ \arccos \lg_{\frac12} x \ge 0 \end{cases} \iff \frac12 \le x \le 2$

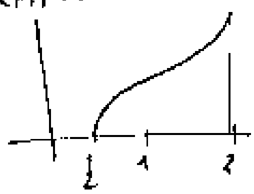
IMM. E FZ. INVERSA

$\sqrt{\arccos \lg_{\frac12} x} = K \ge 0 \iff \arccos \lg_{\frac12} x = K^2 \in [0,\pi] \iff$

$\lg_{\frac12} x = \cos(K^2) \iff x = \left(\frac12\right)^{\cos K^2}$

$\mathrm{Im}\, f = [0, \sqrt{\pi}]$ $\quad f^{-1}(K) = \left(\frac12\right)^{\cos K^2}$

GRAFICO



2. Poichè $x^2 - x + 2 > 0 \ \forall x$ $(\Delta<0)$, la successione è ben definita (e positiva).

$x_{n+1} \ge x_n \iff x_n \le 2$

$x_n > 2$ per induzione

Dunque la successione è decrescente ed il suo limite (che esiste per il teorema delle successioni monotone) è il punto fisso trovato.

In conclusione: $\max = \sup = 4$, $\min$ non $\exists$, $\lim = \inf = 2$

3. $f(x) \sim \dfrac{2\lg x}{\lg x} \to 2$

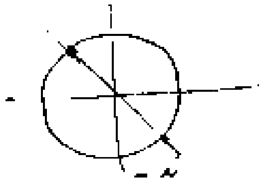
$\left|\dfrac{2\lg x - 1}{\lg x - 1} - 2\right| = \dfrac{1}{|\lg x - 1|} = \dfrac{1}{1-\lg x} < \varepsilon \iff 1-\lg x > \frac1\varepsilon \iff 0<x<e^{1-\frac1\varepsilon}$

posso supporre $0<x<e$

5. L'eq. si risolve nella forma $z^2-(1+i)z+3i=0$, $z\neq 1$, ovvero $iz-$

Applicando la consueta formula di risoluzione delle eq. del secondo ordine, si trova $z = (1+i \pm \sqrt{10}\sqrt{-i})/2 = [1+i \pm \sqrt{5}(-1+i)]/2 = \dots$

Radici quadrate di $-i$



4. $\operatorname{tg} x + x^2 = x + o(x)$

$\cos\sqrt{x+\operatorname{sen} x} \simeq \cos\sqrt{2x} \sim 1 - x + o(x)$

Numeratore $\sim e^x - 1 + x + o(x) = 2x + o(x)$

$\lg \operatorname{sen} x - \lg \operatorname{tg} 3x \sim \lg x - \lg 3x \to -\lg 3$

limite $= -\dfrac{2}{\lg 3}$.

6. $\cdot\ \forall x\in A,\ f(x) \le L$

$\cdot\ \forall \varepsilon>0,\ \exists \bar{x}\in A:\ f(\bar{x}) > L-\varepsilon$

1.

C.E. $\begin{cases} 1+x>0 \\ -1\le \lg(1+x)\le 1 \\ \arcsin \lg(1+x)\ge 0 \end{cases} \iff 0\le x\le e-1$

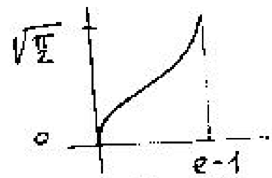
IMM. E FZ. INVERSA

$\sqrt{\arcsin \lg(1+x)} = K \ge 0 \iff \arcsin \lg(1+\operatorname{sen} x) = K^2 \in [0,\pi/2] \iff$

$\lg(1+x) = \operatorname{sen} K^2 \iff x = e^{\operatorname{sen} K^2} - 1.$

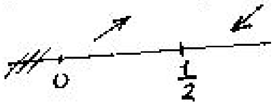
$\operatorname{Im} f = [0, \sqrt{\frac{\pi}{2}}] \qquad f^{-1}(K) = e^{\operatorname{sen} K^2} - 1$

GRAFICO



2. Essendo $x_n>0$, la successione è ben definita (e positiva).

$x_{n+1} \ge x_n \iff x_n \le \frac{1}{2}$



$x_n>2$ per induzione

Successione decrescente.

$\max = \sup = 2$, min non $\exists$, $\inf = \lim = \frac{1}{2}$ (che è il punto fisso).

3. $f(x) \sim \frac{\lg x}{2\lg x} \to \frac{1}{2}$

$\left| \frac{\lg x - 1}{2\lg x - 1} - \frac{1}{2} \right| = \frac{1}{2|2\lg x - 1|} = \frac{1}{2(1-2\lg x)} < \varepsilon \iff 0<x<e^{\frac{1}{2}-\frac{1}{4\varepsilon}}.$

posso suppore $0<x<\sqrt{e}$

4. $\operatorname{sen} 2x + x^2 = 2x + o(x)$ $\qquad$ limite $= 3/\lg 2$

$\cos\sqrt{x+\operatorname{tg} x} \sim \cos\sqrt{2x} \sim 1-x$

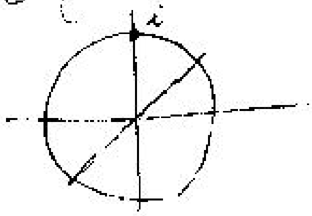
Numeratore $\sim e^{2x} - 1 + x + o(x) = 3x + o(x)$

$\lg \operatorname{sen} 2x - \lg \operatorname{tg} x \sim \lg 2x - \lg x \to \lg 2$

5. $z^2 - 3(1+i)z + 3i = 0$

$z = \frac{3(1+i) \pm \sqrt{6i}}{2} = \frac{3(1+i) \pm \sqrt{3}(1+i)}{2} = \dots$

Radici quadrate di i:

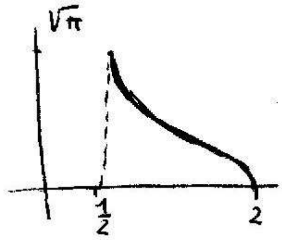


6. . $\forall x \in A$, $f(x) \ge l$

. $\forall \varepsilon>0$, $\exists \bar{x} \in A : f(\bar{x}) < l + \varepsilon$.

1. C.E. $\begin{cases} x \geq 0 \\ -1 \leq \lg_2 x \leq 1 \\ \arccos \lg_2 x \geq 0 \end{cases} \iff \frac{1}{2} \leq x \leq 1$

GRAFICO

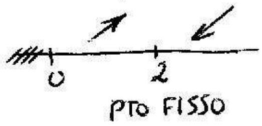


IMM. E INVERSA

$\sqrt{\arccos \lg_2 x} = K \geq 0 \iff \arccos \lg_2 x = K^2 \in [0, \pi] \iff$
$\lg_2 x = \cos K^2 \iff x = 2^{\cos K^2}$
$\operatorname{Im} f = [0, \sqrt{\pi}]$, $f^{-1}(K) = 2^{\cos K^2}$

2. Poichè $x_n^2 - x + 2 > 0 \ \forall x$ $(\Delta < 0)$, la successione è ben definita (e positiva).

$x_{n+1} \geq x_n \iff x_n \leq 2$



$x_n < 2$ per induzione

Dunque la successione è crescente e il suo limite (che esiste per il teorema delle successioni monotone) è il punto fisso trovato.
In conclusione: min = inf = 1, sup = lim = 2, max non esiste

3. $f(x) \sim \dfrac{2 \lg x}{\lg x} \to 2$

$\left| \dfrac{2\lg x + 1}{\lg x + 1} - 2 \right| = \dfrac{1}{|\lg x + 1|} \underset{\substack{\text{posso supporre} \\ x < e^{-1}}}{=} \dfrac{1}{-\lg x - 1} < \varepsilon \iff -\lg x - 1 > \dfrac{1}{\varepsilon}$

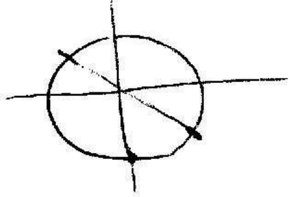
$\iff 0 < x < e^{-1 + \frac{1}{\varepsilon}}$

4. $\cos 2x - \cos x = 1 - 2x^2 - 1 + \frac{1}{2}x^2 + o(x^2) = -\frac{3}{2}x^2 + o(x^2)$
$\operatorname{sen} 2x - \operatorname{sen} x = 2x - x + o(x) = x + o(x)$
$\sqrt{1 + \operatorname{sen} 2x} - e^{-\operatorname{tg} x} = \sqrt{1 + 2x + o(x)} - e^{-x + o(x)} = (1+x) - (1-x) + o(x) =$
$= 2x + o(x)$

$f(x) \sim \dfrac{-\frac{3}{2}x^2}{2x^2} \to -\dfrac{3}{4}$

5. $4z^2 - 2(1+i)z + 3i = 0$ $\qquad z = \dfrac{1+i \pm \sqrt{-10i}}{4} = \dfrac{1+i \pm \sqrt{5}(-1+i)}{4} = \dots$

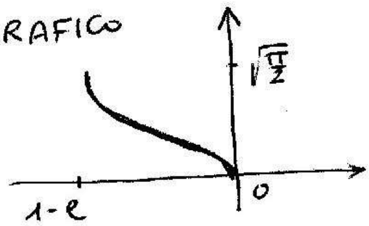
Radici quadrate di $-i$



6. . $\exists\, \bar{x} \in A : f(\bar{x}) = M$
. $\forall x \in A,\ f(x) \leq M$.

1. C.E. $\begin{cases} 1-x>0 \\ -1 \leq \lg(1-x) \leq 1 \\ \lg(1-x) \geq 0 \end{cases} \iff 1-e \leq x \leq 0$

GRAFICO

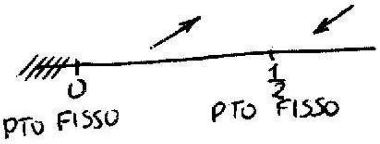


IMM. E INVERSA

$$\sqrt{\operatorname{arcsen} \lg(1-x)} = K \geq 0 \iff \operatorname{arcsen} \lg(1-x) = K^2 \in \left[0, \tfrac{\pi}{2}\right] \iff$$
$$\lg(1-x) = \operatorname{sen} K^2 \iff x = 1 - e^{\operatorname{sen} K^2}$$
$$\operatorname{Im} f = \left[0, \sqrt{\tfrac{\pi}{2}}\right], \quad f^{-1}(K) = 1 - e^{\operatorname{sen} K^2}.$$

2. Successione ben definita e positiva.

$x_{n+1} \geq x_n \iff \sqrt{\dfrac{x_n}{1+2x_n}} \geq x_n \iff \dfrac{x_n}{1+2x_n} \geq x_n^2 \iff x_n\left(2x_n^2 + x_n - 1\right) \leq 0$



$x_n < \frac{1}{2}$ per induzione. Successione crescente; il suo limite (che esiste per il teor. delle successioni monotone) è il punto fisso $\frac{1}{2}$.

In conclusione: $\min = \inf = \frac{1}{4}$, $\sup = \lim = \frac{1}{2}$, max non $\exists$.

3. $f(x) \sim \dfrac{\lg x}{2 \lg x} \to \dfrac{1}{2}$

$$\left| \frac{\lg x + 1}{2 \lg x + 1} - \frac{1}{2} \right| = \underbrace{\frac{1}{|2 \lg x + 1|}}_{\substack{\text{posso supporre} \\ x < e^{-1/2}}} = \frac{1}{-2\lg x - 1} < \varepsilon \iff 0 < x < e^{-\frac{1}{2} - \frac{1}{2\varepsilon}}$$

4. $(1-\cos\sqrt{x})^2 + x \operatorname{sen}\sqrt{x} \sim \left(\frac{1}{2}x + o(x)\right) + \left(x\sqrt{x} + o(x^{3/2})\right) \sim \frac{1}{2}x$

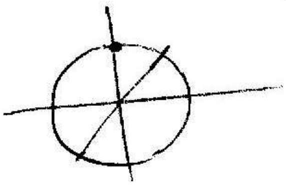
$e^x - (1+x)^{-1} = (1 + x + o(x)) - (1 - x + o(x)) = 2x + o(x)$

$f(x) \sim \dfrac{\frac{1}{2}x}{2x^2} \to +\infty$

5. $4z^2 - 6(1+i)z + 3i = 0$

$z = \dfrac{3 + 3i \pm \sqrt{6}\,\sqrt{i}}{4} = \dfrac{3 + 3i \pm \sqrt{3}\,(1+i)}{4} = \cdots$

Radici quadrate di i



6.
- $\exists\, \bar{x} \in A: \ f(\bar{x}) = m$
- $\forall x \in A, \ f(x) \geq m.$