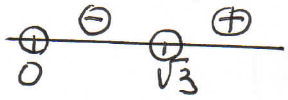
Soluzioni [1]

1. C.E. $\mathbb{R}$. La fz. è dispari: basta studiarla per $x \geq 0$.

SGN



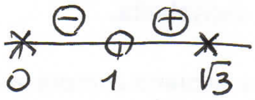
LIM $f(0) = f(\sqrt{3}) = 0$

per $x \to +\infty$ $\quad f(x) \sim x \to +\infty$

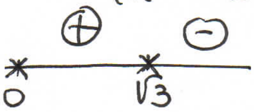
$$f(x) - x = x\left(\sqrt[3]{1-\frac{3}{x^2}} - 1\right) \sim x\left(-\frac{1}{x^2}\right) = -\frac{1}{x} \to 0^-$$

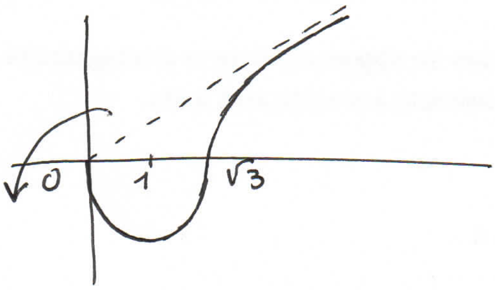
$y = x$ as. obliquo; la fz. sta sotto l'asintoto

DRV $\quad f'(x) = \dfrac{x^2-1}{(x^3-3x)^{2/3}}$, $\quad x \neq 0,\ x \neq \sqrt{3}$ (punti a tg. verticale)



DRV^2 $\quad f''(x) = \dfrac{-2(x^2+1)}{(x^3-3x)^{5/3}}$





L'area richiesta non esiste finita perchè per $x \to +\infty$ $\quad f(x) - x \sim -\frac{1}{x}$.

2. $$\int f(x)\,dx = 2x \operatorname{arctg}\frac{1-x}{1+x} + \int \frac{2x}{1+x^2}\,dx =$$

$$= 2x \operatorname{arctg}\frac{1-x}{1+x} + \lg(x^2+1) + c = \phi(x) + c$$

L'integrale $\int_{-2}^{1} f(x)\,dx$ esiste finito perchè la fz. ha un solo punto di discontinuità in $x=-1$, che è una disc. di I specie.

$$\int_{-2}^{1} f(x)\,dx = \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx = \Big[\phi(x)\Big]_{-2}^{-1^-} + \Big[\phi(x)\Big]_{-1^+}^{1}$$

$$= 2\pi - 4 \operatorname{arctg} 3 + \lg 2/5 .$$

3. $x \in \mathbb{R}$, $y \neq 0$, NO SLZ. COST.

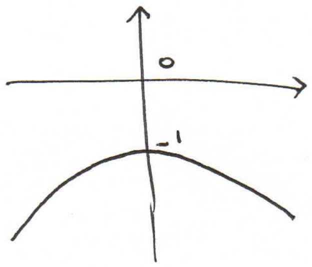
Tenendo conto della C.I., si studia per $y<0$.

$$\int \frac{y}{y^2+1}\,dy = \int x\,dx \Rightarrow \frac{1}{2}\lg(y^2+1) = \frac{1}{2}x^2 + \frac{1}{2}c$$

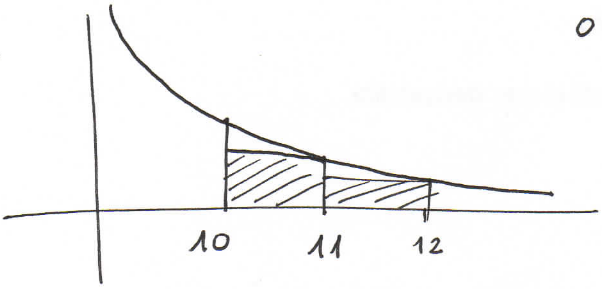
$$y^2+1 = K e^{x^2} \ (K = e^c) \Rightarrow y = -\sqrt{K e^{x^2} - 1} .$$

Imponendo le C.I., si trova che deve essere $K = 2$.

$y = -\sqrt{2e^{x^2}-1}$ $\quad 2e^{x^2} \geq 1 \rightarrow x^2 \geq \ln\frac{1}{2} \rightarrow x \in \mathbb{R}$



4.



$$0 < E < \int_{10}^{+\infty} \frac{1}{x^3} = \frac{1}{2 \cdot 10^2} = 0{,}5 \cdot 10^{-2}$$

Soluzioni [2]

Si deducono con facilità da quelle sopra riportate.