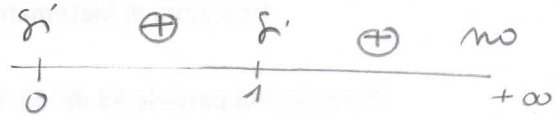
Soluzioni

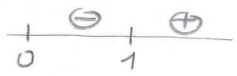
1. $f(t) = \dfrac{t(t-1)}{\lg t}$



$F(x) = \int_1^x f(t)\,dt$

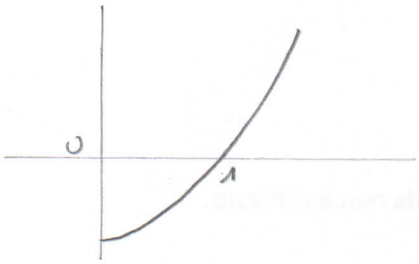
C.E. $x \geq 0$

SGN



$F(1) = 0$

$F(0) = c < 0$



LIM per $x \to +\infty$ $F(x) \to +\infty$

DRV $F'(x) = \dfrac{x(x-1)}{\lg x} > 0$ $\quad x \neq 0,\ x \neq 1$

per $x \to 0$ $\quad F'(x) \to 0$

$x \to 1$ $\quad F'(x) \to 1$

$x \to +\infty$ $\quad F'(x) \to +\infty$ NO ASINTOTO

2. $\displaystyle\int \frac{dx}{(x-1)\sqrt{1-(x-1)^2}}\,dx = \int \frac{dt}{\operatorname{sen} t} = \int \frac{ds}{s} = \lg|s| + C = \dots$

$x - 1 = \operatorname{sen} t$

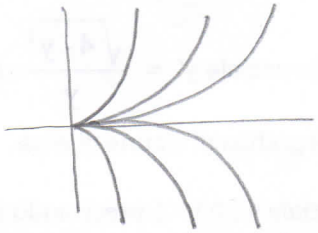
$dx = \cos t\,dt$

$\lg \frac{t}{2} = s$

3. C.E. $x > 0$, $y \in \mathbb{R}$ $\qquad y = 0$ sol. costante

$$\int \frac{dy}{y(1+y^2)} = \int \frac{dx}{x} \to \frac{1}{2}\lg\frac{y^2}{y^2+1} = \lg x + c \to \frac{|y|}{\sqrt{y^2+1}} = Ke^x \quad (K>0)$$

$$y = \pm\frac{Kx}{\sqrt{1-K^2x^2}}, \quad 0 < x < \frac{1}{K}$$



4. Per la periodicità si studia – ad esempio – in $[0, 2\pi]$.

CE: $x \in [0, \pi/2) \cup [\frac{3}{2}\pi, 2\pi]$

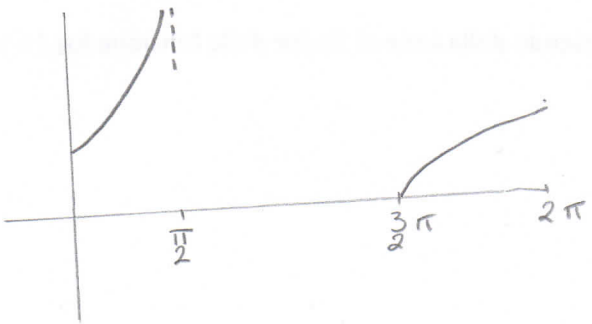
SGN positivo

$$\lim_{x\to\frac{\pi}{2}^-} \frac{\sqrt{\cos x}}{1-\operatorname{sen} x} = \lim_{x\to\frac{\pi}{2}^-} \frac{\sqrt{\cos x}\,(1+\operatorname{sen} x)}{\cos^2 x} = +\infty$$

DRV $f'(x) = \dfrac{2+\operatorname{sen} x}{2\sqrt{\cos x}\,(1-\operatorname{sen} x)}$ sempre positiva

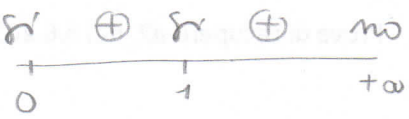
per $x \to 0^+$ $\quad f'(x) \to 1$

per $x \to \frac{3}{2}\pi^+$ $\quad f'(x) \to +\infty$



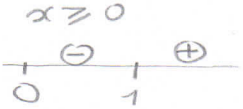
Soluzioni [2]

1. $f(t) = \sqrt{\dfrac{\lg t}{t(t-1)}}$



$F(x) = \int_1^x f(t)\,dt$

C.E. $x \geq 0$ $\qquad F(1) = 0$

SGN 

$\qquad F(0) = c < 0$

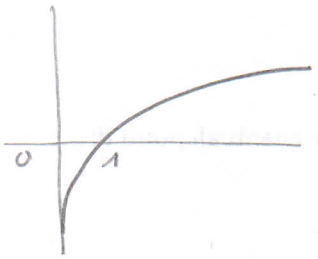
LIM per $x \to +\infty$ $\quad F(x) \to +\infty$

DRV $F'(x) = \sqrt{\dfrac{\lg x}{x(x-1)}} > 0 \quad x \neq 0,\ x \neq 1$

per $x \to 0$ $\quad F'(x) \to +\infty$

$x \to 1$ $\quad F'(x) \to 1$

$x \to +\infty$ $\quad F'(x) \to 0$ NO ASINTOTO



2. $\displaystyle\int \frac{dx}{(x+1)\sqrt{-2x-x^2}} = \int \frac{dx}{(x+1)\sqrt{1-(x+1)^2}} = \int \frac{dt}{\operatorname{sen} t} = \int \frac{1}{s}\,ds = \lg|s| + c \ldots$

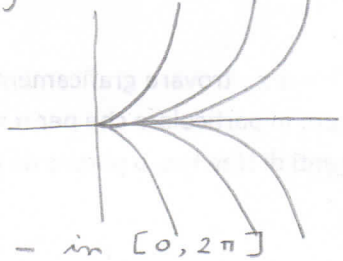
$x+1 = \operatorname{sen} t$ $\qquad \operatorname{tg}\frac{t}{2} = s$

3. C.E. $x > 0$ $\qquad y = 0$ sol costante

$y \in \mathbb{R}$

$$\int \frac{dy}{y(1+4y^2)} = \int \frac{dx}{x} \to \frac{1}{2}\lg\frac{y^2}{1+4y^2} = \lg x + c \to \frac{|y|}{\sqrt{1+4y^2}} = kx \quad (k>0)$$

$$y = \pm\frac{kx}{\sqrt{1-4k^2x^2}}, \quad 0 < x < \frac{1}{2k}$$



4. Per la periodicità si studia – ad es. – in $[0, 2\pi]$

C.E. $x \in [0, \pi/2] \cup (\frac{3}{2}\pi, 2\pi]$

SGN positiva

$$\lim_{x \to \frac{3}{2}\pi^+} \frac{\sqrt{\cos x}}{1+\operatorname{sen} x} = \lim_{x \to \frac{3}{2}\pi^+} \frac{\sqrt{\cos x}\,(1-\operatorname{sen} x)}{\cos^2 x} = +\infty$$

DRV $f'(x) = \dfrac{\operatorname{sen} x - 2}{2\sqrt{\cos x}\,(1+\operatorname{sen} x)} < 0$

per $x \to 0^+$ $\quad f'(x) \to -1$

$x \to \frac{\pi}{2}^-$ $\quad f'(x) \to -\infty$