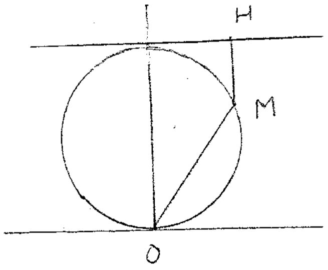
1.



$$\begin{cases} x^2+y^2-2y=0 \\ y=mx \end{cases} \longrightarrow (m^2+1)x^2-2mx=0$$

$$M=\left(\frac{2m}{m^2+1}, \frac{2m^2}{m^2+1}\right) \qquad H=\left(\frac{2m}{m^2+1}, 2\right)$$

$$OM^2+MH^2=\frac{4m^2+4m^4}{(m^2+1)^2}+\left(2-\frac{2m^2}{m^2+1}\right)^2=4\,\frac{m^4+m^2+1}{(m^2+1)^2}$$

La derivata della fz. si annulla per $m=\pm 1$, che forniscono le due rette di minimo.

2.

$$\operatorname{arctg} x = x-\frac{x^3}{3}+o(x^4)$$
$$\lg\left(1+x^2-\frac{x^4}{3}+o(x^4)\right)=\left(x^2-\frac{x^4}{3}\right)-\frac{1}{2}x^4+o(x^4)=x^2-\frac{5}{6}x^4+o(x^4)$$
$$e^{x^2}=1+x^2+\frac{x^4}{2}+o(x^4)$$
$$\sqrt{1-2x^2}=1+\frac{1}{2}(-2x^2)-\frac{1}{8}(4x^4)+o(x^4)=1-x^2-\frac{x^4}{2}+o(x^4)$$
$$\cos^2 x=\left(1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^4)\right)^2=1+\frac{x^4}{4}-x^2+\frac{x^4}{12}+o(x^4)=1-x^2+\frac{x^4}{3}+o(x^4)$$
$$f(x)\sim -\frac{4}{3}x^4\Big/-\frac{5}{6}x^4 \longrightarrow \frac{8}{5}$$

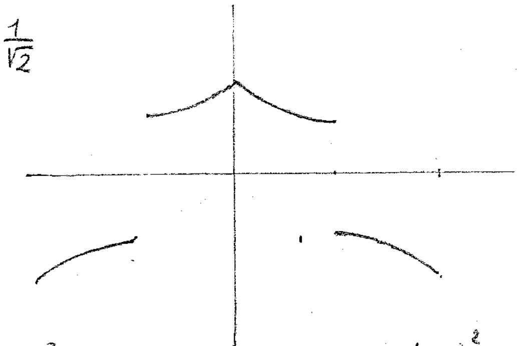
3.

periodica di periodo $2\pi \to$ si studia in $[-\pi,\pi]$
pari $\to$ si studia in $[0,\pi]$
simm. $(\frac{\pi}{2},0) \to$ si studia in $[0,\pi/2]$.

$$f(0)=1,\quad \lim_{x\to\frac{\pi}{2}^-} f(x)=\lim_{x\to\frac{\pi}{2}^-}\frac{\sqrt{1-\operatorname{sen}x}\,\sqrt{1+\operatorname{sen}x}}{\cos x\,\sqrt{1+\operatorname{sen}x}}=\lim_{x\to\frac{\pi}{2}^-}\frac{\cos x}{\sqrt{2}\cos x}=\frac{1}{\sqrt{2}}$$

$$f'(x)=-\frac{(1-\operatorname{sen}x)^{3/2}}{2\cos^2 x} \qquad f'(0)=-\frac{1}{2},\quad \lim_{x\to\frac{\pi}{2}^-} f'(x)=0$$

$$f''(x)=\frac{\operatorname{sen}^2 x-4\operatorname{sen}x+3}{4\cos^3 x}(1-\operatorname{sen}x)^{1/2}=\frac{(1-\operatorname{sen}x)^{3/2}(3-\operatorname{sen}x)}{4\cos^3 x}$$



Punti di disc.: $\frac{\pi}{2}+k\pi$, punti di non deriv. $k\pi$

Scegliamo l'intervallo $[0,\pi/2]$ $\quad (f:[0,\pi/2]\to[1/\sqrt{2},1])$

$$\frac{\sqrt{1-\operatorname{sen}x}}{\cos x}=\alpha \Rightarrow 1-\operatorname{sen}x=\alpha^2\cos^2 x \Rightarrow 1-\operatorname{sen}x=\alpha^2-\alpha^2\operatorname{sen}^2 x \Rightarrow x=\operatorname{arcsen}\frac{1-\alpha^2}{\alpha^2}$$

$$f(x)=\frac{\sqrt{1-x+x^3/6+o(x^3)}}{1-\frac{1}{2}x^2+o(x^3)}=\left(1+\frac{1}{2}\left(-x+\frac{x^3}{6}\right)-\frac{1}{8}x^2-\frac{1}{16}x^3+o(x^3)\right)\left(1+\frac{1}{2}x^2+o(x^3)\right)$$
$$=1-\frac{1}{2}x+\frac{3}{8}x^2-\frac{11}{48}x^3+o(x^3)$$

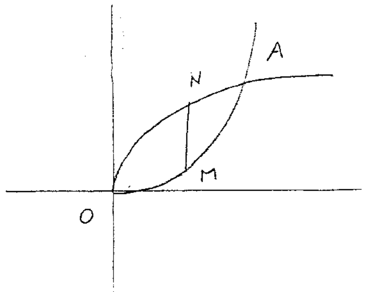
4. $\quad x+x^2\lg x\sim x \qquad x^2\lg x \qquad \sqrt{1+x^2}-\sqrt[3]{1+x^2}\sim\frac{x^2}{6} \qquad \frac{x-\operatorname{sen}x+x^6}{\sqrt{x}}\sim\frac{x^{5/2}}{6}$

5. $$\lg(1+x)=\sum_{k=1}^{3}(-1)^{k+1}\frac{x^k}{k}-\frac{1}{4(1+\xi)^4}x^4$$

Essendo $x=-\frac{1}{10}$, $-\frac{1}{10}<\xi<0$.
Errore negativo ($\to$ appross. per eccesso)
$$|E|=\frac{1}{4(1+\xi)10^4}<\frac{1}{36}10^{-3}<3\cdot 10^{-4}$$

$$\lg\frac{9}{10}\sim-\frac{1}{10}-\frac{1}{2}\left(\frac{1}{100}\right)-\frac{1}{3}\left(\frac{1}{1000}\right)=-0{,}105\cdots$$

1.



A (1,1)
M (x, x^2)
N $(x, \sqrt{x})$
O (0,0)
$x \in [0,1]$

area $= \frac{1}{2} x (\sqrt{x} - x^2) + \frac{1}{2}(1-x)(\sqrt{x} - x^2) =$
$= \frac{1}{2}(\sqrt{x} - x^2)$

Massima per $x = \left(\frac{1}{2}\right)^{4/3}$

2. $\lg(1-x^2) = -x^2 - \frac{x^4}{2} + o(x^4)$

$\sqrt{1+x^2} = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + o(x^4)$

$\lg(1-x^2) - 2 + 2\sqrt{1+x^2} = -\frac{3}{4}x^4 + o(x^4)$

$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^4)$

$e^{x^2} - 3 + 2\cos x = \frac{7}{24}x^4 + o(x^4)$

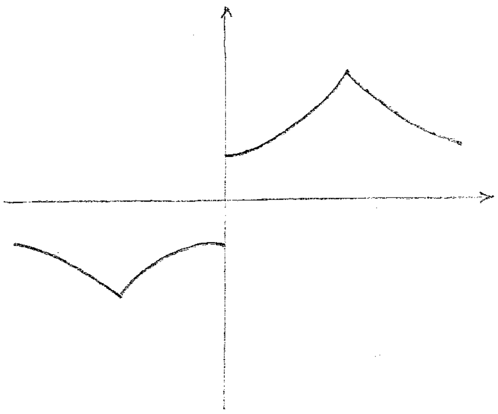
limite $= -\frac{18}{7}$

3. periodica di periodo 2π $\rightarrow$ si studia in $[\pi, \pi]$
dispari $\rightarrow$ si studia in $[0, \pi]$
simm. retta $x = \pi/2$ $\rightarrow$ si studia in $[0, \pi/2]$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x/\sqrt{2}}{x} = 1/\sqrt{2}$ $\qquad f(\pi/2) = 1$

$f'(x) = \frac{(1-\cos x)^{3/2}}{2 \operatorname{sen}^2 x}$; $\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \frac{\left(\frac{1}{2}x^2\right)^{3/2}}{2x^2} = 0$; $f'(\frac{\pi}{2}) = \frac{1}{2}$

$f''(x) = \frac{(3-\cos x)(1-\cos x)^{3/2}}{4 \operatorname{sen}^3 x}$



Punti discont. $k\pi$, di non deriv. $\frac{\pi}{2} + k\pi$

scegliamo $[0, \pi/2]$ $(f: [0, \pi/2] \rightarrow [1/\sqrt{2}, 1])$.

$\frac{\sqrt{1-\cos x}}{\operatorname{sen} x} = \alpha \Rightarrow x = \arccos \frac{1-\alpha^2}{\alpha^2}$

$f(x) = \frac{x - x^3/6 + o(x^3)}{\sqrt{3 - \frac{x^2}{2} + o(x^3)}} = \frac{x - x^3/6 + o(x^3)}{\sqrt{3}\sqrt{1 - \frac{x^2}{6} + o(x^3)}} = \frac{1}{\sqrt{3}}\left(x - \frac{x^3}{6} + o(x^3)\right)\left(1 + \frac{x^2}{12} + o(x^3)\right) = \frac{1}{\sqrt{3}}\left(x - \frac{x^3}{12}\right) + o$

4. $\frac{\operatorname{tg}^2 x}{\sqrt[3]{x}} \sim x^{5/3}$ $\qquad \sqrt{4+2x^2} - 2 \sim \frac{x^2}{2}$ $\qquad \frac{e^{-x^2/2} - \cos x}{x^{3/2}} \sim \frac{x^{5/2}}{12}$ $\qquad x^3 - x^4 \lg x \sim x^3$

5 $\sqrt[3]{999} = 10\sqrt[3]{1 - \frac{1}{1000}}$

$\sqrt[3]{1+x} = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \frac{10}{243}\frac{x^4}{(1+\xi)^{11/3}}$

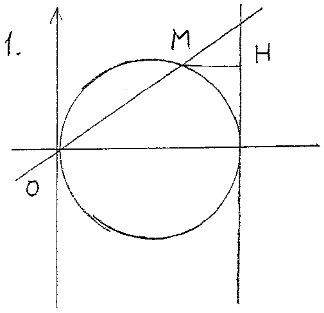
$x = -1/1000$
$\xi \in (-\frac{1}{1000}, 0)$

Errore negativo ($\rightarrow$ appross. per eccesso)

$|E| < \frac{10}{243}\frac{1}{10^{12}}\left(\frac{1000}{999}\right)^{11/3} < \frac{10}{200}\frac{1}{10^{12}}\left(\frac{1000}{999}\right)^4 = \frac{1}{2 \cdot 10 \cdot 999^4} \sim 5 \cdot 10^{-14}$

$\sqrt[3]{999} \simeq 9.99666555\ldots$

1.



$$\begin{cases} x^2+y^2-2x=0 \\ y=mx \end{cases} \rightarrow (m^2+1)x^2-2x=0$$

$$M=\left(\frac{2}{m^2+1}, \frac{2m}{m^2+1}\right) \qquad H=\left(2, \frac{2m}{m^2+1}\right)$$

$$OM^2+MH^2 = 4\,\frac{m^2+m+1}{(m^2+1)^2}$$

La derivata si annulla per $m=\pm 1$, che forniscono le due rette di minimo.

2.

$$\operatorname{sen}^2 x = x^2 - \frac{x^4}{3} + o(x^4)$$

$$\sqrt[3]{1+\operatorname{sen}^2 x} = 1+\frac{1}{3}x^2 - \frac{2}{9}x^4 + o(x^4)$$

$$3\sqrt[3]{1+\operatorname{sen}^2 x} - 5 + 2\cos x = -\frac{7}{12}x^4 + o(x^4)$$

$$\sqrt{1-x^2} = 1-\frac{1}{2}x^2-\frac{1}{8}x^4+o(x^4)$$

$$\sqrt{1-x^2}+x^2/2-1 = -\frac{1}{8}x^4+o(x^4)$$

limite $= 14/3$.

3.

Periodica di periodo $2\pi \rightarrow$ si studia in $[-\pi,\pi]$

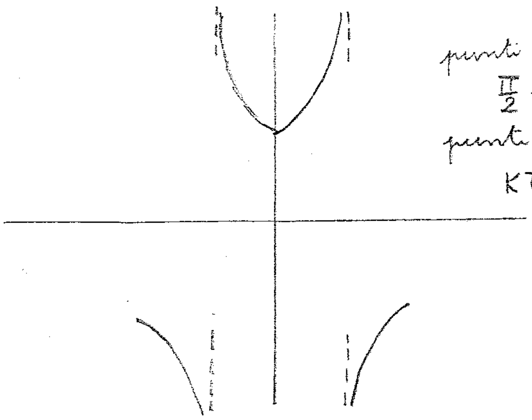
Pari $\rightarrow$ si studia in $[0,\pi]$

Simm. $(\pi/2, 0) \rightarrow$ si studia in $[0,\pi/2]$

$f(0)=1$, $\lim_{x\to\frac{\pi}{2}^-} f(x) = +\infty$

$$f'(x) = \frac{(1+\operatorname{sen} x)^{3/2}}{2\cos x} \qquad f'(0)=\frac{1}{2}$$

$$f''(x) = \frac{(1+\operatorname{sen} x)^{3/2}(3-\operatorname{sen} x)}{4\cos^2 x}$$



punti di discont.
$\frac{\pi}{2}+K\pi$

punti non deriv.
$K\pi$

Scegliamo l'intervallo $[0,\pi/2)$ $\left(f: [0,\pi/2) \rightarrow [1,+\infty)\right)$

$$\frac{\sqrt{1+\operatorname{sen} x}}{\cos x} = \alpha \Rightarrow x = \operatorname{arcsen}\frac{\alpha^2-1}{\alpha^2}$$

$$\frac{\sqrt{1+\operatorname{sen} x}}{\cos x} = \frac{\sqrt{1+x-\frac{x^3}{6}+o(x^3)}}{1-\frac{1}{2}x^2+o(x^3)} = \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{48}x^3+o(x^3)\right)\left(1+\frac{1}{2}x^2+o(x^3)\right) =$$

$$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{11}{48}x^3+o(x^3)$$

4.

$$\sqrt{x}+x^2\lg x \sim \sqrt{x} \qquad x^2\lg x \qquad \sqrt[3]{1+x^2}-\sqrt[4]{1+x^2} \sim \frac{x^2}{12} \qquad \frac{x^2-\operatorname{sen}^2 x+x^6}{\sqrt{x}} \sim \frac{x^{7/2}}{3}$$

5.

$$\lg(1+x) = \sum_{K=1}^{3}(-1)^{K+1}\frac{x^K}{K} - \frac{1}{4(1+\xi)^4}x^4$$

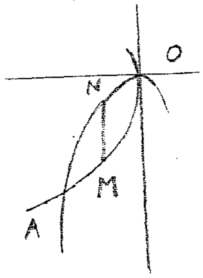
Essendo $x=-\frac{1}{5}$, $-\frac{1}{5}<\xi<0$

Errore negativo ($\rightarrow$ appross. per eccesso)

$$|E| = \frac{1}{4(1+\xi)^4 5^4} < \frac{1}{4(4/5)^4 5^4} = \frac{1}{4\cdot 4^4} < 10^{-3}$$

$$\lg\frac{8}{10} \simeq -\frac{1}{5}-\frac{1}{2}\,\frac{1}{25}-\frac{1}{3}\,\frac{1}{125} \sim -0,222$$

1.



$A=(-1,-1)$
$M=(x,-\sqrt{-x})$
$N=(x,-x^2)$
$O=(0,0)$
$x\in[-1,0]$

area $=\frac{1}{2}(x+1)(\sqrt{-x}-x^2)+\frac{1}{2}(-x)(\sqrt{-x}-x^2)=\frac{1}{2}(\sqrt{-x}-x^2)$

massima per $x=-1/\sqrt[3]{16}$

2.

$\lg(1+x^2)=x^2-x^4/2+o(x^4)$
$\sqrt{1-x^2}=1-x^2/2-x^4/8+o(x^4)$
$\lg(1+x^2)-2+2\sqrt{1-x^2}=-\frac{3}{4}x^4+o(x^4)$

$e^{x^2}=1+x^2+x^4/2+o(x^4)$
$\cos 2x=1-2x^2+2/3\,x^4+o(x^4)$
$e^{x^2}-2+\cos 2x=-x^2+\frac{7}{6}x^4+o(x^4)$

limite $=0$

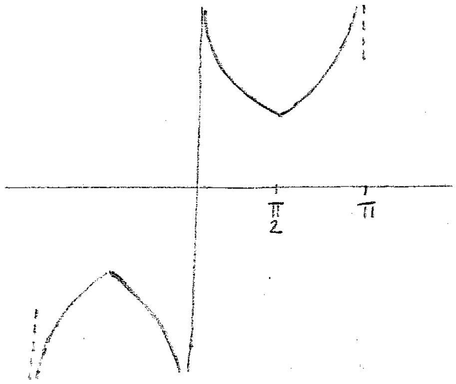
3.

periodica di periodo $2\pi\to$ si studia in $[-\pi,\pi]$
dispari $\to$ si studia in $[0,\pi]$
simm. retta $x=\pi/2\to$ si studia in $[0,\pi/2]$

$\lim_{x\to 0^+} f(x)=+\infty$, $f(\pi/2)=1$

$f'(x)=-\frac{(1+\cos x)^{3/2}}{2\,\mathrm{sen}^2 x}$, $f'(\pi/2)=-1/2$

$f''(x)=\frac{(\cos x+3)(1+\cos x)^{3/2}}{4\,\mathrm{sen}^3 x}$



punti discontinuità
$k\pi$
punti non derivabilità
$\frac{\pi}{2}+k\pi$

Scegliamo l'intervallo $[0,\pi/2]$
$f:(0,\pi/2]\to[1,+\infty)$

$\frac{\sqrt{1+\cos x}}{\mathrm{sen}\, x}=\alpha\to x=\arccos\frac{\alpha^2-1}{\alpha^2}$

$\frac{\mathrm{sen}\, x}{\sqrt{1+\cos x}}=\frac{x-x^3/6+o(x^3)}{\sqrt{2-\frac{1}{2}x^2+o(x^3)}}=\frac{x-x^3/6+o(x^3)}{\sqrt{2}\sqrt{1-\frac{1}{4}x^2+o(x^3)}}=\frac{1}{\sqrt{2}}(x-x^3/6+o^3)(1+\frac{1}{8}x^2+o^3)=\frac{x-x^3/24}{\sqrt{2}}+$

4.

$\frac{\mathrm{arctg}^2 x}{\sqrt[3]{x}}\sim x^{5/3}$ $\quad\sqrt[3]{8+2x^2}-2\sim\frac{x^2}{6}$ $\quad\frac{\mathrm{tg}(x^2)-\mathrm{sen}^2 x}{x^{3/2}}\sim\frac{x^{5/2}}{3}$ $\quad x^3-x^4\lg x\sim x^3$

5.

$\sqrt[3]{990}=10\sqrt[3]{1-\frac{1}{100}}$

$\sqrt[3]{1+x}=1+\frac{1}{3}x-\frac{1}{9}x^2+\frac{5}{81}x^3-\frac{10}{243}\frac{x^4}{(1+\xi)^{11/3}}$ $\quad x=-\frac{1}{100}$, $\xi\in(-\frac{1}{100},0)$

Errore negativo ($\to$ appross. per eccesso)

$|E|<\frac{10}{243}\frac{1}{10^8}\left(\frac{100}{99}\right)^{11/3}<\frac{10}{200}\frac{1}{10^8}\left(\frac{100}{99}\right)^4=\frac{1}{2\cdot 10\cdot 99^4}\sim 5,2\ 10^{-10}$

$\sqrt[3]{990}\sim 9,96655\ldots$