Soluzioni

1. Eq. differenziale a variabili separate.

C.E. $x \neq 0$, $y \geq 1$

$y = 1$ soluzione costante

Per trovare le altre variabili si procede come di consueto:

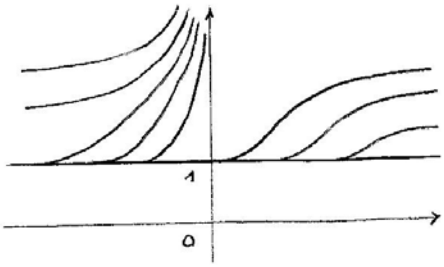
$$\int \frac{dy}{2\sqrt{y-1}} = \int \frac{dx}{x^2} \Rightarrow \sqrt{y-1} = c - \frac{1}{x}$$

$y = 1 + \left(c - \frac{1}{x}\right)^2$ sotto la condizione $c - \frac{1}{x} > 0$.

$\frac{cx - 1}{x} > 0$ per $c > 0$ $x \in (-\infty, 0) \cup (1/c, +\infty)$

per $c = 0$ $x \in (-\infty, 0)$

per $c < 0$ $x \in (1/c, 0)$



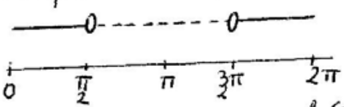
2. $f(x) = 2\cos x\,(\operatorname{sen} x + 1)$

C.E. $\mathbb{R}$

fz. 2π-periodica $\rightarrow$ si studia in $[0, 2\pi]$

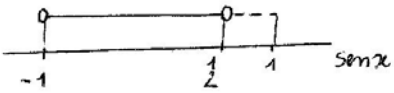
né pari né dispari

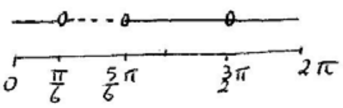
SGN



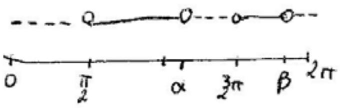
LIM $f(0) = f(2\pi) = 2$, $f(\pi) = -2$

DRV $f'(x) = -2(2\operatorname{sen}^2 x + \operatorname{sen} x - 1)$

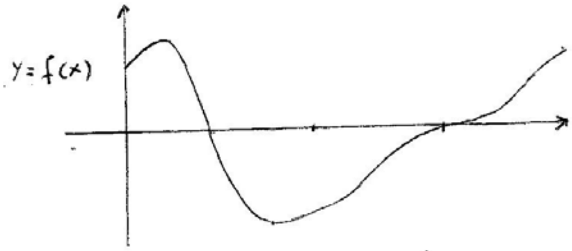


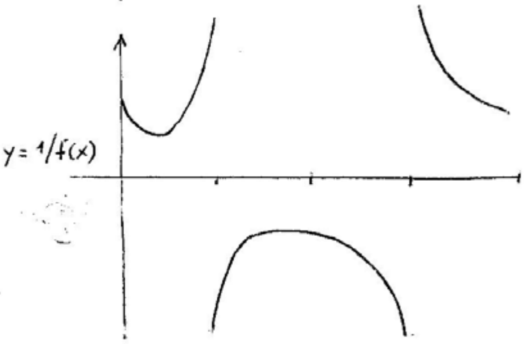


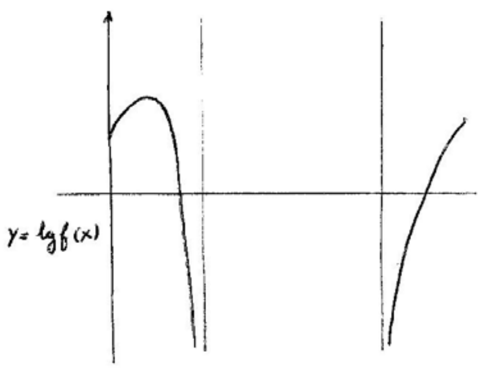
DRV^2 $f''(x) = -2\cos x\,(4\operatorname{sen} x + 1)$



$\alpha = \pi + \arcsin\frac{1}{4}$, $\beta = 2\pi - \arcsin\frac{1}{4}$.







3. Si pone $tg\frac{x}{2} = t$:

$$\int \frac{4}{(t+1)^2(t^2+1)}\,dt \underset{\text{Hermite}}{=} \int \left(\frac{2}{t+1} + \frac{2}{(t+1)^2} - \frac{2t}{t^2+1} \right) dt =$$

$$= 2 \lg|t+1| - \frac{2}{t+1} - \lg(t^2+1) + c$$

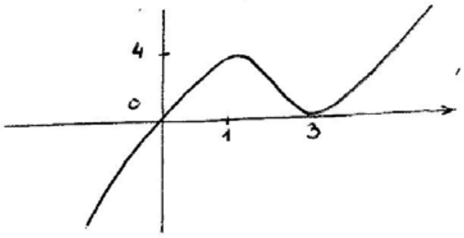
$$= 2 \lg \frac{(tg\frac{x}{2}+1)^2}{1+tg^2\frac{x}{2}} - \frac{2}{tg\frac{x}{2}+1} + c =$$

$$= 2 \lg(1+ sen\, x) - \frac{2}{tg\frac{x}{2}+1} + c$$

4. L'integrale è improprio a causa dell'estremo $-\pi/2$ in cui la fz. diverge.

$sen x = -sen\pi + \cos\pi\,(x+\pi) + o(x+\pi) = -(x+\pi) + o(x+\pi)$

$f(x) \sim \frac{1}{x+\pi}$: è un infinito di ordine 1; l'integrale non esiste.



5. $f(x) = x^3 - 6x^2 + 9x = x(x-3)^2$

$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

L'eq. data si può scrivere nella forma $f(x) = -k$.

La retta $y = -k$ interseca il grafico in un solo punto se $-k<0$ oppure $-k>4$, cioè se $k<-4$ o $k>0$.