Introduzione alla Matematica

1.

CE. $\begin{cases} 2x-1>0 \\ \dfrac{1}{\sqrt{2x-1}} \leq 1 \end{cases} \iff 2x-1 \geq 1 \iff x \geq 1$

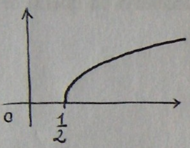
$\arcsin \dfrac{1}{\sqrt{2x-1}} = \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \iff \dfrac{1}{\sqrt{2x-1}} = \operatorname{sen} \alpha \iff$

$\dfrac{1}{2x-1} = \operatorname{sen}^2 \alpha, \ \alpha \in \left[0, \frac{\pi}{2}\right] \iff x = \frac{1}{2}\left(1 + \frac{1}{\operatorname{sen}^2 \alpha}\right), \ \alpha \in \left(0, \frac{\pi}{2}\right]$

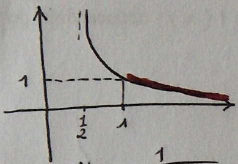
$\operatorname{Im} f = (0, \pi/2]$

$f^{-1}(\alpha) = \frac{1}{2}\left(1 + \frac{1}{\operatorname{sen}^2 \alpha}\right)$

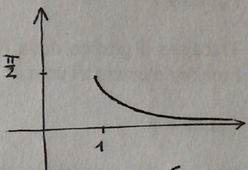
2.



$y = \sqrt{2x-1}$



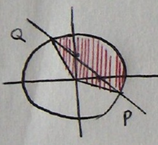
$y = \dfrac{1}{\sqrt{2x-1}}$



$y = f(x)$

3.

(i) $\begin{cases} X + Y > \frac{1}{2} \\ X^2 + Y^2 = 1 \end{cases}$



$P\left(\frac{1+\sqrt{7}}{4}, \frac{1-\sqrt{7}}{4}\right)$

$Q\left(\frac{1-\sqrt{7}}{4}, \frac{1+\sqrt{7}}{4}\right)$

$\arcsin \frac{1-\sqrt{7}}{4} + 2k\pi < x < \arcsin \frac{1+\sqrt{7}}{4} + 2k\pi$

(ii) CE. : $x \neq \pi + 2k\pi$ e $x \neq \frac{\pi}{2} + 2k\pi$

Nel CE il denominatore è sempre >0.
Studiamo il segno del numeratore, ponendo $\operatorname{tg} \frac{x}{2} = t$: il numeratore diventa $3t - \sqrt{3} \geq 0$ per $t \geq \frac{\sqrt{3}}{3}$ cioè $\frac{\pi}{6} + k\pi \leq \frac{x}{2} < \frac{\pi}{2} + k\pi$ cioè $\frac{\pi}{3} + 2k\pi \leq x < \pi + 2k\pi$.