Principali formule di Taylor di punto iniziale $x_0 = 0$

$$e^x = \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + o(x^n)$$

$$\operatorname{sen}x = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$(1+x)^\alpha = \sum_{k=0}^{n} \binom{\alpha}{k} x^k + o(x^n)$$

$$= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \ldots + \frac{\alpha(\alpha-1)\ldots(\alpha-(n-1))}{3!} x^n + o(x^n)$$

In generale:

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\ldots(\alpha-(k-1))}{k!} \quad , \text{ se } k \neq 0$$

$$\binom{\alpha}{0} = 1$$

$$\frac{1}{1+x} = \sum_{k=0}^{n} (-1)^k x^k + o(x^n) = 1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + o(x^n)$$

$$\frac{1}{1-x} = \sum_{k=0}^{n} x^k + o(x^n) = 1 + x + x^2 + x^3 + \ldots + x^n + o(x^n)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \sum_{k=3}^{n} (-1)^{k-1} \frac{1 \cdot 3 \cdot 5 \cdots (2k-3)}{2 \cdot 4 \cdot 6 \cdots (2k)} x^k + o(x^n)$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3}{8}x^2 + \sum_{k=3}^{n} (-1)^k \frac{1.3.5....(2k-1)}{2.4.6...(2k)} x^k + o(x^n)$$

$$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{x^2}{2} + \frac{3}{8}x^4 + \sum_{k=3}^{n} \frac{1.3.5....(2k-1)}{2.4.6...(2k)} x^{2k} + o(x^{2n+1})$$

$$\frac{1}{1+x^2} = \sum_{k=0}^{n} (-1)^k \; x^{2k} + o(x^{2n+1})$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + (-1)^n \frac{x^n}{n!} + o(x^n)$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \ldots + (-1)^n \frac{x^{2n}}{n!} + o(x^{2n+1})$$

$$a^x = e^{x \log a} = 1 + x \log a + \frac{x^2}{2!} \log^2 a + \ldots + \frac{x^n}{n!} \log^n a + o(x^n)$$

$$\cos x = \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\log(1+x) = \sum_{k=1}^{n} (-1)^{k+1} \frac{x^k}{k} + o(x^n)$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n+1} \frac{x^n}{n} + o(x^n)$$

$$\log_a(1+x) = \frac{\log(1+x)}{\log a} = \frac{1}{\log a}\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n+1} \frac{x^n}{n}\right) + o(x^n)$$

arctg x = x - x^3 / 3 + x^5 / 5 + ……

arcsen x = x + x^3 / 6 + 3 x^5 / 40 + …..

arcos x = π/2 - x - x^3 / 6 - 3 x^5 / 40 + …..

$$\text{tg}\, x = x + x^3/3 + 2x^5/15 + o(x^5).$$