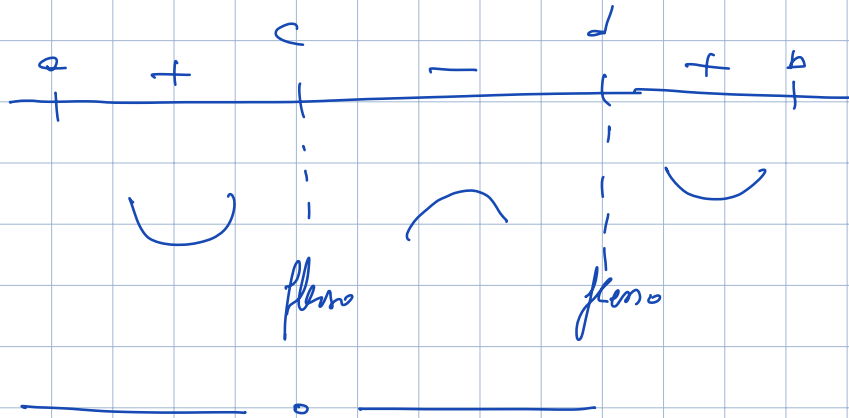


Ricaviamo 3/6/24

$$f: (a,b) \rightarrow \mathbb{R}$$

$$f'(x)$$

$$f''(x)$$



2 parti, coppia con 3 punti

Scelte una ordered di 3 oggetti tra 12

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220$$



Operazioni che non cambiano il det

- Sostituire una riga o colonna con se stessa + multiplo altro

con coeff. 1

$$\det \begin{pmatrix} 2 & -1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & -3 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{pmatrix} = +3 \cdot \det \begin{pmatrix} 2 & -1 & 1 \\ 0 & -3 & 2 \\ 1 & 1 & -1 \end{pmatrix} + 1 \cdot \det \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & -3 & 2 \end{pmatrix}$$

$$\det \begin{pmatrix} 2 & -1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & -3 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{pmatrix} = \det \begin{pmatrix} 2 & -1 & 1 & 0 \\ -2 & -2 & 3 & 0 \\ 0 & -3 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

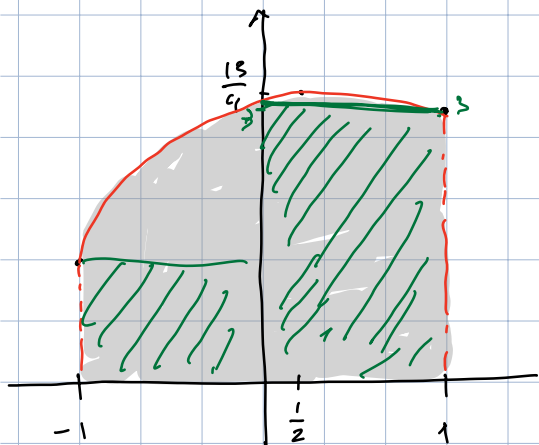
_____ 0 _____

$$\begin{cases} 9x - 7y + 5z = 2 \\ 4x + 2y - z = 7 \\ x + 3y + 6z = -4 \end{cases}$$

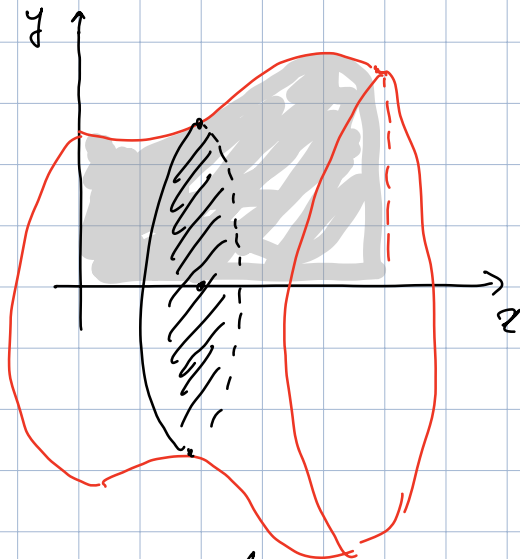
$$x = \dots \quad y = \frac{\begin{vmatrix} 2 & 2 & 7 \\ 7 & -1 & -4 \\ 1 & -1 & 6 \end{vmatrix}}{\begin{vmatrix} 9 & -7 & 5 \\ 4 & 2 & -1 \\ 1 & 3 & 6 \end{vmatrix}} \quad z = \dots$$

$$\begin{pmatrix} 9 & -7 & 5 \\ 4 & 2 & -1 \\ 1 & 3 & 6 \end{pmatrix}^{-1} = \frac{1}{\det(\dots)} \cdot \begin{pmatrix} 21 & 123 & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

$$f: [-1, 1] \rightarrow \mathbb{R} \quad f(x) = 3 + x - x^2$$



$$f: [0, 1] \rightarrow \mathbb{R} \quad f(x) = x + e^x$$



$$\text{Vol (solid, rotat)} = \pi \cdot \int_0^1 f(x)^2 dx$$

Base di W^\perp dove $W = \text{Span} \left(\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)$

$$W^\perp: \begin{cases} x - y + u = 0 \\ 2x + y + z - u = 0 \end{cases} \quad \begin{cases} y = x - u \\ 3x + z = 0 \end{cases} \quad \begin{cases} z = -3x \\ y = x - u \end{cases}$$

$\dim(W) = 2$

$\mathbb{R}^4 = W \oplus W^\perp \Rightarrow \dim(W^\perp) = 4 - 2 = 2$

Base $W^\perp = \begin{pmatrix} 1 \\ 1 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

$W = \text{Span} \left(\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)$

Calcolo base di W^\perp del tipo

$$\begin{pmatrix} -1 \\ -1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

Base di W^\perp , $W = \text{Span} \left(\begin{pmatrix} 2 \\ 1 \\ 7 \\ -4 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 6 \\ 8 \end{pmatrix} \right)$

Mi servono due vettori ortog. ai due dati.

Ne dico uno con scala componente nulla:

$$\begin{pmatrix} 30 \\ 0 \\ -28 \\ 9 \end{pmatrix}$$

$$f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$$f(e_1) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad f(e_2) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad f(2e_3) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(So che se $A \in M_{m \times n}$, $A: \mathbb{R}^n \rightarrow \mathbb{R}^m$,
 $A \cdot e_j = j$ -esima colonna di A)

f è l'applicazione associata a

$$\begin{pmatrix} 2 & 1 & \frac{1}{2} \\ -1 & 1 & 0 \end{pmatrix}$$

$$(A) \quad \text{Ker}(f): \begin{cases} 2x + y + \frac{1}{2}z = 0 \\ -x + y = 0 \end{cases}$$

$$\begin{cases} y = x \\ z = -6x \end{cases}$$

$$\begin{pmatrix} 1 \\ 1 \\ -6 \end{pmatrix}$$

$$(B) \dim(\text{Im}(f)) = \dim(\mathbb{R}^3) - \dim(\text{Ker}(f)) \\ = 3 - 1 = 2$$

(C) injective : no ; surjective : no

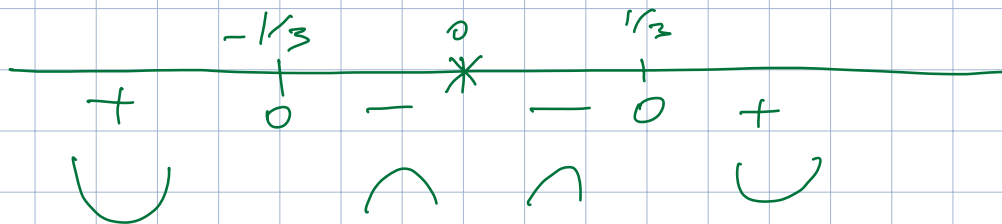
$$f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$$

$$f(x) = x^2 - \frac{27}{x^2}$$

$$f'(x) = 2x - 27(-2) \cdot x^{-3}$$

$$f''(x) = 2 - 27(-2) \cdot (-3) \cdot x^{-4} \\ = 2(1 - 81x^{-4})$$

$$f''(x) = 0 \text{ pour } 1 - \frac{81}{x^4} = 0 \quad x^4 = \frac{1}{3^4} \quad x = \pm \frac{1}{3}$$



convexe sur $(-\infty, -1/3] \cup [1/3, +\infty)$

concave sur $[-1/3, 0) \cup (0, 1/3]$

Esibiru bera ortogonal di $W: 3x - 2y + z = 0$.

Pemilihan bera gramian & ortogonalisasi

$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \rightarrow u_1 = \frac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

v_1 v_2

$$\tilde{u}_2 = v_2 - v_2 \cdot u_1 \cdot u_1$$

$$= \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} - \frac{-1}{\sqrt{11}} \cdot \frac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \frac{1}{11} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

~~$$= \frac{1}{11} \begin{pmatrix} 3 \\ 12 \\ 21 \end{pmatrix}$$~~

$$u_2 = \frac{1}{\sqrt{3+144+441}} \cdot \begin{pmatrix} 3 \\ 12 \\ 21 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} \rightarrow u_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

v_1 v_2

$$\tilde{u}_2 = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} - \frac{6}{5} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} -5 \\ -6 \\ 3 \end{pmatrix} \quad u_2 = \frac{1}{\sqrt{25+36+9}} \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}$$

$W: 3x - 2y + z = 0$

$$\begin{pmatrix} 17 \\ 14 \\ -23 \end{pmatrix}, \begin{pmatrix} \vdots \\ \vdots \end{pmatrix}$$

————— 0 —————

$$\lim_{x \rightarrow +\infty} \sin\left(\frac{3}{x}\right) \tan\left(\frac{2}{x}\right)$$

$$t = \frac{3}{x} \quad t \rightarrow 0^+$$

$$\lim_{t \rightarrow 0^+} \sin(t) \tan\left(\frac{2}{3}t\right)$$

$$= \exp\left(\lim_{t \rightarrow 0^+} \underbrace{\tan\left(\frac{2}{3}t\right)}_{\frac{2}{3}t} \cdot \log\left(\underbrace{\sin(t)}_t\right)\right)$$

$$= \exp\left(\lim_{t \rightarrow 0^+} \frac{2}{3} \cdot t \cdot \log(t)\right) = \exp(0) = 1$$