Ist. Mat. I - CIA
29/11/23

(110) Trovare approx Taylor IV in 0 per
$$f(x) = \log(1+\cos(x))$$

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{6!}x^6 + \dots + o(x^{2n})$$

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots + \underbrace{o(x^n)}_{x \to 0}$$

Sbagliato sostituire la approx di $\cos(x)$ in x in quella di $\log(1+x)$.

(Invece se era $\log(1+\sin(x))$ era ok).

Dunque devo per forza calcolare
$f(0), f'(0), f''(0) \dots$

$$\rightsquigarrow f(x) = \sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!} x^k + o(x^4)$$

(47) $\sqrt{x}\cdot\log(x)$

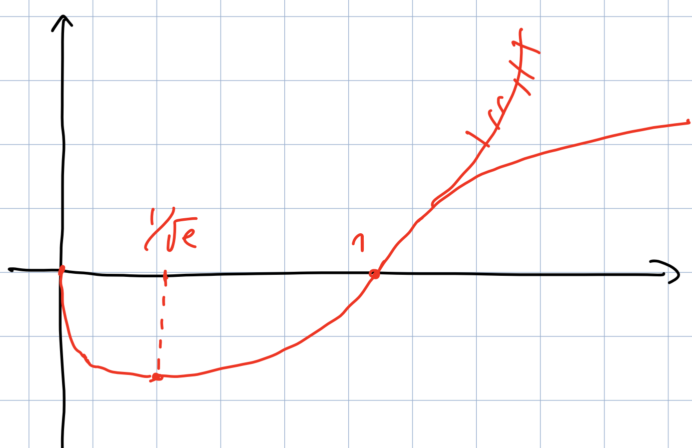
$D=(0,+\infty)$; nulla per $x=1$; neg $x<1$; pos. $x>1$.

$\lim_{x\to 0^+} f(x) = 0$; $\lim_{x\to+\infty} = +\infty$; $\lim \frac{f(x)}{x} = 0$

$$f'(x) = \frac{1}{2\sqrt{x}}\cdot\log(x) + \sqrt{x}\cdot\frac{1}{x} = \frac{1}{\sqrt{x}}\left(\frac{1}{2}\log(x)+1\right)$$

$f'(x)=0$ per $x=\frac{1}{\sqrt{e}}$

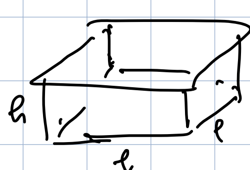
$$\lim_{x\to 0^+} f'(x) = (+\infty)\cdot(-\infty) = -\infty$$



$$f''(x) = -\frac{1}{2}\cdot\frac{1}{x\sqrt{x}}\left(\frac{1}{2}\log(x)+1\right) + \frac{1}{\sqrt{x}}\cdot\frac{1}{2x}$$

$$= \frac{1}{2x\sqrt{x}}\left(\frac{1}{2}\log(x)+2\right)$$

nulla in $x=$ ..

(51) Max vol. di scatola  di area 108?

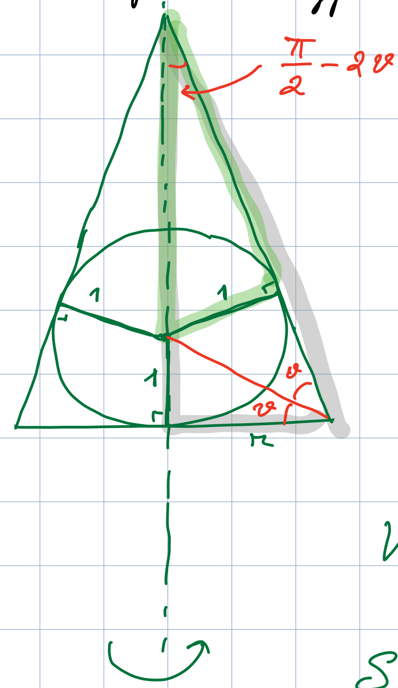
$A = l^2+4lh$ $\qquad h = \frac{108-l^2}{4l} = \frac{27}{l}-\frac{l}{4}$

$V = l^2 h = 27l - \frac{1}{4}l^3$ $\qquad 0\le l\le\sqrt{108} = 6\sqrt{3}$

$V=0$ agli estremi

$V' = 27-\frac{3}{4}l^2$ $\qquad V'=0$ per $l^2 = 27/3\cdot 4$, $l=6$.

(52) Qual è il massimo per area/volume di cono circoscritto a sfera di raggio 1.



$$r = \frac{\cos(\theta)}{\sin(\theta)}$$

$$h = 1 + \frac{1}{\sin\left(\frac{\pi}{2} - 2\theta\right)}$$

$$a = r + \frac{\cos\left(\frac{\pi}{2} - 2\theta\right)}{\sin\left(\frac{\pi}{2} - 2\theta\right)}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$S = \pi r^2 + \frac{1}{2}(2\pi r) \cdot a$$

Fatto: max raggiunto per $\sin(\theta) = \frac{1}{\sqrt{3}}$ nei due casi.

10/1/23

(1) $K = \mathbb{Q} / \mathbb{R}$; $X = \{x \in K : x^3 < 2\}$
X inf/sup lim?
ha inf o sup in K?

$-k \in X \quad \forall k \in \mathbb{N} \Rightarrow$ non è inf. lim.
$2^3 > 2 \Rightarrow \forall x \in X$ si ha $x < 2 \Rightarrow$ è sup. lim.
In $\mathbb{R}$ certamente ha sup. s e $s^3 = 2$.

Poiché $\nexists\, y \in \mathbb{Q}$ t.c. $y^3 = 2$ ho $s \notin \mathbb{Q}$
quindi su $\mathbb{Q}$ X non ha sup.

② Lancio 10 volte moneta; escono 4 T + 6 C.
Quante sequenze T/C?

1	2	3	4	5	6	7	8	9	10
C	C	T	C	T	T	C	C	T	C

$$\binom{10}{4} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = \frac{\overset{10\cdot 9\cdot 8\cdot 7}{\cancel{10!}}}{4! \cdot \cancel{6!}}$$

$= 210$

③ Moltepl. di i come radice di
$$z^4 + (1-3i)z^3 - (5+4i)z^2 + 5(i-1)z + 2(1+i)$$

molt. $= m$ per z_0 come radice di $p(z)$ se
$$p(z) = (z - z_0)^m \cdot q(z) \quad \text{con } q(z_0) \neq 0.$$

	1	$1-3i$	$-5-4i$	$-5+5i$	$2+2i$
i		i	$2+i$	$3-3i$	$-2-2i$
	1	$1-2i$	$-3-3i$	$-2+2i$	✓
i		i	$+1+i$	$2-2i$	
	1	$1-i$	$-2-2i$	✓	
i		i	i		
	1	1	X		

$m = 2$

④ $$\lim_{x \to 0} \frac{(\sin(x) - x) \cdot \arctan(x)}{(1 - \cos(x)) \cdot \log(1 + 2x^2)}$$ $\frac{0 \cdot 0}{0 \cdot 0}$

$$\frac{-\frac{1}{6}x^3 \cdot x}{\frac{1}{2}x^2 \cdot 2x^2} = -\frac{1}{6}$$

⑤ $f(x) = \sqrt[6]{x^5} - \sqrt[4]{1 - x^3}$. Def su $D = \ldots$?

$f'(x) = ?$ def su ?

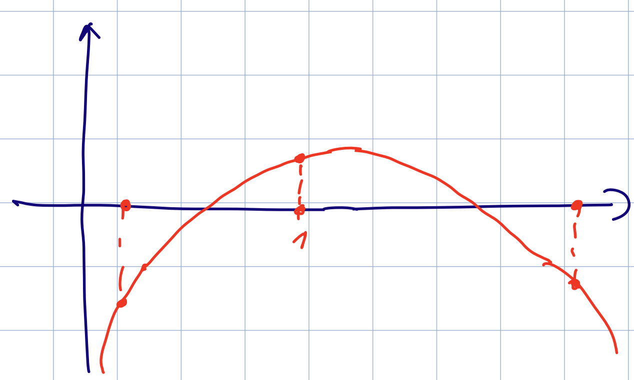
$$D: \begin{cases} x^5 \geq 0 \\ 1 - x^3 \geq 0 \end{cases} \qquad \begin{cases} x \geq 0 \\ x \leq 1 \end{cases} \qquad D = [0, 1]$$

$$f'(x) = \frac{5}{6} x^{-\frac{1}{6}} - \frac{1}{4}(1 - x^3)^{-\frac{3}{4}} \cdot (-3x^2) \quad \text{su } (0, 1)$$

⑥ $f: (0, +\infty) \to \mathbb{R}$ $\quad f(x) = 2 - x + \log(x)$

verificare che ha almeno due zeri.

$$\lim_{x \to 0} f(x) = -\infty \qquad \lim_{x \to +\infty} f(x) = -\infty$$



$f(1) = 2 - 1 + 0 = 1 > 0$

⑦ $\lim_{x\to+\infty}\left(\sin\left(\frac{3}{x}\right)\right)^{\tan\left(\frac{2}{x}\right)}$ $\qquad 0^0$

$$= \exp \lim_{x\to\infty} \log\left(\left(\sin\left(\frac{3}{x}\right)\right)^{\tan\left(\frac{2}{x}\right)}\right)$$

$$= \exp\left(\lim_{x\to\infty} \tan\frac{2}{x}\cdot\log\left(\sin\left(\frac{3}{x}\right)\right)\right) \qquad \frac{3}{x}=t$$

$$= \exp\left(\lim_{t\to 0} \underbrace{\frac{2}{3}\underset{0}{t}\cdot\underbrace{\log(\sin(t))}_{-\infty}}_{0}\right) = \exp(0) = 1$$

sì

⑧ $f(x) = x^2 - \frac{27}{x^2}$ su $x\neq 0$; intervalli conc/conv.

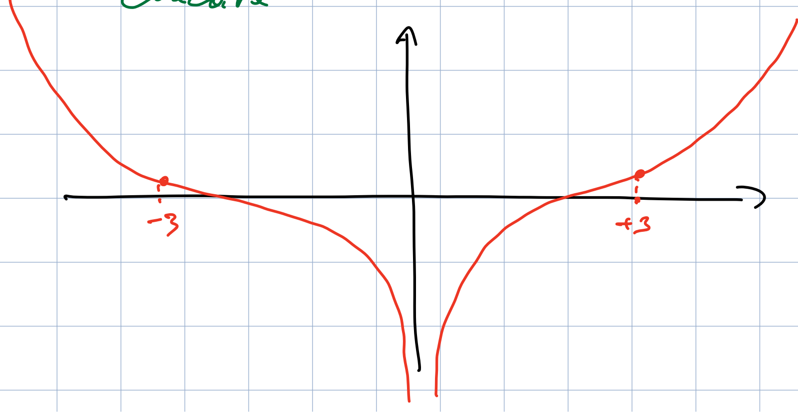
$f'(x) = 2x - 27\cdot(-2)\cdot x^{-3} = 2x + 54x^{-3}$ concorde con x

$f''(x) = 2 + 2\cdot 27\cdot(-3)x^{-4} = 2(1-81x^{-4})$

		-3		0		3	
f''	$+$		$-$		$-$		$+$
	$\cup$		$\cap$		$\cap$		$\cup$

Convessa negli intervalli contenuti in $(-\infty,-3]$ e $[3,+\infty)$
Concava " " " $\underbrace{[-3,0) \text{ e } (0,3]}$

~~$[-3,3]$~~



$\boxed{25/11/22}$

(3) $\sqrt{3}\cdot z^2 + (\sqrt{2}+i\sqrt{3})\cdot z + i\sqrt{2} = 0$

$$\Delta = (\sqrt{2}+i\sqrt{3})^2 - 4\sqrt{3}\cdot(i\sqrt{2})$$

$$= 2 + 2i\sqrt{6} - 3 - 4i\sqrt{6}$$

$$= 2 - 2i\sqrt{6} - 3 = (\sqrt{2} - i\sqrt{3})^2$$

$$z_{1,2} = \frac{-(\sqrt{2}+i\sqrt{3}) \pm (\sqrt{2}-i\sqrt{3})}{2\sqrt{3}} = \begin{cases} -i \\ \sqrt{\frac{2}{3}} \end{cases}$$

—— o ——

Foglio 2 - Ese 3 - (a)

$f: \mathbb{Z} \to \mathbb{N}, \quad f(n) = |n|$

ammette inversa sinistra o destra? Se sì esibirle.

$$\begin{tikzcd} X \arrow[r, bend left, "g"] & Y \arrow[l, bend left, "h"] \end{tikzcd}$$

$h \circ g = id_X$

h inversa sin. di g

g inversa dx di h

g iniettiva, h surgettiva.

f surgettiva non iniettiva

Inversa destra è $t: \mathbb{N} \to \mathbb{Z}$ t.c.

$$\mathbb{N} \xrightarrow{t} \mathbb{Z} \xrightarrow{f} \mathbb{N}$$

posso scegliere $t(n) = n$

(o anche $t(n) = -n$)

(o anche $t(n) = (-1)^n \cdot n$)

$$f \circ t = \mathbb{N}$$