IST MAT I - CIA 1/12/2022

ZANICHELLI P. 169 (4-40)

5) CALCOLARE LA DERIVATA DELLE SEGUENTI FUNZIONI.

$$f(x) = \log|x| = \begin{cases} \log x & x > 0 \\ \log(-x) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x} & x > 0 \\ \frac{1}{x} & x < 0 \end{cases} = \frac{1}{x}$$

$$f(x) = \log(-x) \qquad \left.\begin{aligned} g(x) &= -x \\ h(x) &= \log x \end{aligned}\right\} h(g(x))$$

$$(h(g(x)))' = h'(g(x)) \cdot g'(x)$$

$$= \frac{1}{g(x)} \cdot -1 = \frac{1}{-x} \cdot -1 = \frac{1}{x}$$

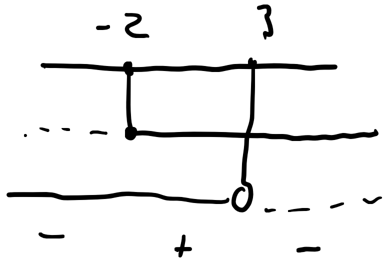
$$f(x) = \log(3x)$$

$$f'(x) = \frac{1}{\cancel{3}x} \cdot \cancel{3} = \frac{1}{x}$$

$$f(x) = \log\left|\frac{x+2}{3-x}\right|$$

$$\left|\frac{x+2}{3-x}\right| = \begin{cases} \dfrac{x+2}{3-x} & \dfrac{x+2}{3-x} \geq 0 \\[2ex] \dfrac{x+2}{x-3} & \dfrac{x+2}{3-x} < 0 \end{cases} = \begin{cases} \dfrac{x+2}{3-x} & -2 \leq x < 3 \\[2ex] \dfrac{x+2}{x-3} & x < -2 \vee x > 3 \end{cases}$$

$$\frac{x+2}{3-x} \geq 0 \Rightarrow \begin{array}{ll} x+2 \geq 0, & x \geq -2 \\ 3-x > 0 & x < 3 \end{array}$$



$$\Updownarrow$$

$$-2 \leq x < 3$$

$$\log\left|\frac{x+2}{3-x}\right| = \begin{cases} \log\left(\dfrac{x+2}{3-x}\right) & -2 \leq x < 3 \\[2ex] \log\left(\dfrac{x+2}{x-3}\right) & x < -2 \vee x > 3 \end{cases}$$

$$3 - \cancel{x} + \cancel{x} + 2$$

$$\left(\log\left(\frac{x+2}{3-x}\right)\right)' = \frac{3-x}{x+2} \cdot \frac{1\cdot(3-x) - (-1)\cdot(x+2)}{(3-x)^2} = \frac{\cancel{3-x}}{x+2} \cdot \frac{5}{(3-x)^{\cancel{2}}}$$

$$= \frac{5}{(x+2)(3-x)}$$

$$\left(\log\left(\frac{x+2}{x-3}\right)\right)' = \left(\log(x+2) - \log(x-3)\right)' = \frac{1}{x+2} - \frac{1}{x-3}$$

$$= \frac{\cancel{x} - 3 - \cancel{x} - 2}{(x+2)(x-3)} = -\frac{5}{(x+2)(x-3)}$$

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$$\left(\log\left|\frac{x+2}{3-x}\right|\right)' = \frac{5}{(x+2)(3-x)}$$

6) $f(x) = e^{-3x} \cdot (x^2+2x-1)$

$$f'(x) = \left(e^{-3x}\right)' \cdot (x^2+2x-1) + e^{-3x} \cdot (x^2+2x-1)'$$

$$= -3e^{-3x} \cdot (x^2+2x-1) + e^{-3x}(2x+2)$$

$$= -3e^{-3x}x^2 - 4e^{-3x}x + 5e^{-3x}$$

10) $f(x) = e^{2x}(2\sin 3x - 4\cos 3x)$

$$f'(x) = \left(e^{2x}\right)'(2\sin 3x - 4\cos 3x) + e^{2x}(2\sin 3x - 4\cos 3x)'$$

$$= 2e^{2x}(2\sin 3x - 4\cos 3x) + e^{2x}(2\cos 3x \cdot 3 - 4(-\sin 3x) \cdot 3)$$

$$= 4e^{2x}\sin 3x - 8e^{2x}\cos 3x + 6e^{2x}\cos 3x + 12e^{2x}\sin 3x$$

$$= 16e^{2x}\sin 3x - 2e^{2x}\cos 3x$$

13) $f(x) = 2^{x^2+3x}$

$$f'(x) = 2^{x^2+3x} \cdot \log 2 \cdot (2x+3)$$

17) $f(x) = x^{x\log x} = e^{\log\left(x^{x\log x}\right)}$

$\underbrace{\log(x^c) = c\log x}$

$= e^{x\log x \cdot \log x} = e^{x(\log x)^2}$

$f'(x) = e^{x(\log x)^2} \cdot \left(x(\log x)^2\right)' = e^{x(\log x)^2} \cdot \left(1 \cdot (\log x)^2 + \right.$

$\left. + \cancel{x} \cdot 2\log x \cdot \frac{1}{\cancel{x}}\right)$

$= e^{x(\log x)^2}\left((\log x)^2 + 2\log x\right)$

20) SCRIVERE L'EQUAZIONE DELLA RETTA TANGENTE A $y = f(x)$ IN x_0

$f(x) = \sin x \qquad x_0 = \frac{\pi}{3}$

$$y = f'(x_0)(x - x_0) + f(x_0)$$

$f(x_0) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$f'(x_0) = \cos\frac{\pi}{3} = \frac{1}{2}$

$y = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} = \frac{1}{2}x + \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$

24) $f(x) = \log x \qquad x_0 = 1$

$f(x_0) = \log 1 = 0$

$f'(x_0) = \frac{1}{1} = 1$

$y = (x - 1) + 0$