Ist Mat I - CIA

10/11/2022

Foglio 4

f) $f(x) = x\dfrac{2-e^x}{5+3e^x}$ $\qquad D = \mathbb{R}$

$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

$$\lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{2-e^x}{5+3e^x} \overset{-\frac{\infty}{\infty}}{=} \lim_{x\to\infty}\frac{\cancel{e^x}\left(\overset{0}{\frac{2}{e^x}}-1\right)}{\cancel{e^x}\left(\underset{0}{\frac{5}{e^x}}+3\right)} = -\frac{1}{3} = m$$

$$\lim_{x\to-\infty}\frac{f(x)}{x} = \lim_{x\to-\infty}\frac{2-\overset{0}{e^x}}{5+\underset{0}{3e^x}} = \frac{2}{5}$$

$$\lim_{x\to\infty} f(x)-\left(-\frac{1}{3}\right)x = \lim_{x\to\infty} x\left(\frac{2-e^x}{5+3e^x}+\frac{1}{3}\right) =$$

$$= \lim_{x\to\infty} x\left(\frac{6-\cancel{3e^x}+5+\cancel{3e^x}}{3(5+3e^x)}\right) =$$

$$= \lim_{x\to\infty}\frac{11x}{3(5+3e^x)} = 0$$

PER LE GERARCHIE DEGLI INFINITI

$$\lim_{x\to\infty}\frac{x}{e^x} = 0$$

$\Rightarrow$ PER $x\to\infty$ C'È UN ASINTOTO OBLIQUO, DI EQUAZIONE

$$y = -\frac{1}{3}x$$

$$\lim_{x\to-\infty} f(x) - \frac{2}{5}x = \lim_{x\to-\infty} x\left(\frac{2-e^x}{5+3e^x} - \frac{2}{5}\right)$$

$$= \lim_{x\to-\infty} x\left(\frac{\cancel{10} - 5e^x - \cancel{10} - 6e^x}{5(5+3e^x)}\right) \overset{x\to-\infty \;\; "xe^x"}{=} 0$$

$$\lim_{x\to-\infty} -11xe^x \overset{y=-x}{=} \lim_{y\to+\infty} 11ye^{-y} = \lim_{y\to\infty} \frac{11y}{e^y} = 0$$

$\Rightarrow$ PER $x \to -\infty$ C'È UN ASINTOTO OBLIQUO DI EQUAZIONE

$$y = \frac{2}{5}x$$

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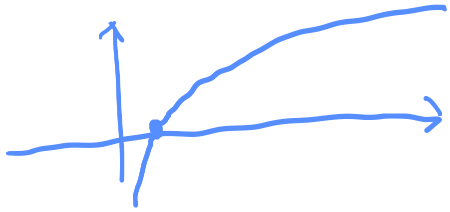
k) $f(x) = 7 + \dfrac{5}{|\log x|}$ $\quad f: D \to \mathbb{R} \quad D = \{x > 0,\ x \neq 1\}$

$$\begin{cases} |\log x| \neq 0 \\ x > 0 \end{cases}$$



$\log 1 = \log e^0 = 0$

$$\lim_{x\to0^+} 7 + \frac{5}{|\log x|} = 7$$

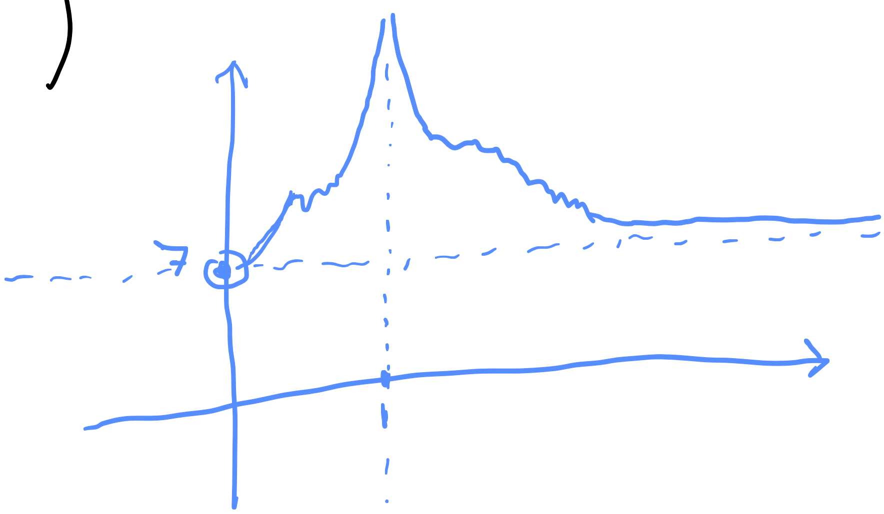
NON C'È ASINTOTO VERTICALE PER $x \to 0^+$

$$\lim_{x\to1^-} 7 + \frac{5}{\underbrace{|\log x|}_{\to 0^+}} = +\infty$$

$$\lim_{x\to1^+} 7 + \frac{5}{|\log x|} = +\infty$$

LA RETTA $x = 1$ È ASINTOTO VERTICALE.

$$\lim_{x\to\infty} 7 + \frac{5}{|\log x|} = 7$$



ZANICHELLI P 135

RICORDO

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\sin 2x}{2x}=1$$

$$\lim_{x\to 0}\frac{\sin(x^3)}{x^3}=1$$

21)

$$\lim_{x\to 0}\frac{x\,(\sin(2x))^2}{\sin(x^3)}=$$

$$\lim_{x\to 0}\frac{(2x)^2}{(2x)^2}\,\frac{x\,(\sin(2x))^2}{\sin(x^3)}=$$

$$\lim_{x\to 0}\left(\frac{\sin(2x)}{2x}\right)^2\cdot 2^2\,\frac{x^3}{\sin(x^3)}=4$$

$$\downarrow \quad 1 \qquad\qquad \downarrow \quad 1$$

$\sin 2x \sim 2x \qquad x\to 0$

23)

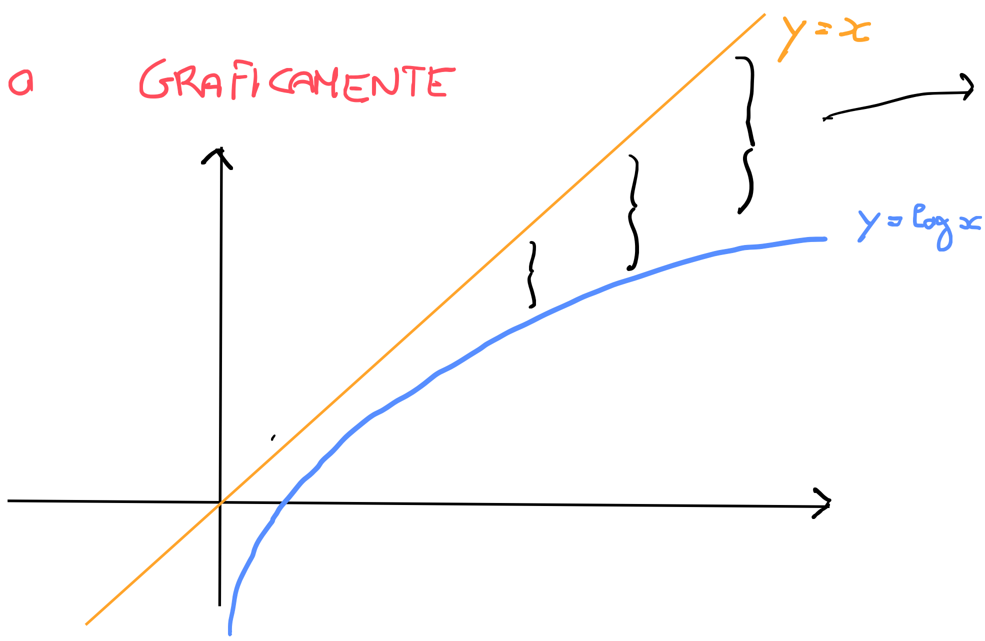
$$\lim_{x\to\infty}\frac{\log(\log x)}{1+\log x}\overset{y=\frac{1}{\log x}}{=}\lim_{y\to 0}\frac{\log(y^{-1})}{1+\frac{1}{y}}$$

$$=\lim_{y\to 0}\frac{-\log(y)}{\frac{y+1}{y}}=\lim_{y\to 0}\frac{-y\log y}{y+1}=0$$

$$\lim_{y\to 0}-y\log y\overset{z=\frac{1}{y}}{=}\lim_{z\to\infty}-\frac{\log(z^{-1})}{z}=\lim_{z\to\infty}\frac{\log(z)}{z}=0$$

$$\lim_{x\to\infty}\frac{\log x}{x}=0$$

GRAFICAMENTE



IL RAPPORTO $\frac{\log x}{x}$ TENDE A 0