FOGLIO 2

ES 10

i) $z^2\bar{z} + 4i|z|^2 + i\bar{z} + 4 = z\bar{z}^2 + iz$

$\underbrace{z^2\bar{z}} + 4i|z|^2 + i\bar{z} + 4 - \underbrace{z\bar{z}^2} - iz = 0$

$z\bar{z}\left(z - \bar{z}\right) + i\left(\bar{z} - z\right) + 4\left(i|z|^2 + 1\right) = 0$

$(z - \bar{z})(z\bar{z} - i) + 4(i|z|^2 - i^2) = 0$

$(z - \bar{z})(|z|^2 - i) + 4i(|z|^2 - i) = 0$

$(|z|^2 - i)(z - \bar{z} + 4i) = 0$

- $|z|^2 - i = 0$? MAI $\quad |z|^2 \in \mathbb{R} \quad i \notin \mathbb{R}$
- $2i\,\Im m(z) + 4i = 0 \Rightarrow 2i\left(\Im m(z) + 2\right) = 0$

SOLUZIONE $\{z \in \mathbb{C} : \Im m(z) = -2\}$ □

ES 11

a) $p(z) = (i-1)z^3 - 5iz^2 + (2+3i)z - 2(i+1) \qquad p(z) \in \mathbb{C}[z]$

$z_1 = -i$ È UNA RADICE ($p(z_1) = 0$) $\iff \underline{p(z) = (z - z_1)\,q(z)}$

DIVIDIAMO $p(z)$ PER $(z - z_1) = (z + i)$

	$i-1$	$-5i$	$2+3i$	$-2(i+1)$
$-i$		$1+i$	$-4-i$	$2+2i$
	$i-1$	$1-4i$	$-2+2i$	//

$$p(z) = (z+i)\left((i-1)z^2 + (1-4i)z + (-2+2i)\right)$$

FORMULA RADICI POLINOMIO DI GRADO 2

$$z_{2,3} = \frac{-1+4i \pm \sqrt{(1-4i)^2 - 4(i-1)(-2+2i)}}{2(i-1)}$$

$$= \frac{-1+4i \pm \sqrt{1-16-8i+\cancel{8}-\cancel{8}+16i}}{2(i-1)}$$

$$= \frac{-1+4i \pm \sqrt{1-16+8i}}{2(i-1)}$$

$$= \frac{-1+4i \pm \sqrt{(1+4i)^2}}{2(i-1)} = \begin{cases} \dfrac{-1+4i+1+4i}{2(i-1)} = z_2 \\[2ex] \dfrac{-1+4i-1-4i}{2(i-1)} = z_3 \end{cases}$$

$$z_2 = \frac{4i}{i-1} \cdot \frac{i+1}{i+1} = \frac{-4+4i}{-2} = \boxed{2-2i}$$

$$z_3 = \frac{-1}{i-1} \cdot \frac{i+1}{i+1} = \frac{-1-i}{-2} = \boxed{\frac{1}{2} + \frac{1}{2}i}$$

$$p(z) = (z+i)\left(z-(2-2i)\right)\left(z-\left(\frac{1}{2}+\frac{1}{2}i\right)\right)$$

□

b) $p(z) = z^3 + (2-4i)z^2 - (4+8i)z - 8$

$z_1 = -2$

$$\begin{array}{c|ccc|c} & 1 & 2-4i & -4-8i & -8 \\ -2 & & -2 & 8i & 8 \\ \hline & 1 & -4i & -4 & // \end{array}$$

$$p(z) = (z+2)(z^2 - 4iz - 4) = (z+2)(z-2i)^3$$

$$1^2 \qquad (-2i)^2$$

$z_1 = -2 \qquad z_2 = z_3 = 2i \qquad$ MOLTEPLICITÀ 2. $\square$

FOGLIO 3

ES 1

a) MONOTONA

- CRESCENTE $\quad (a_n)_{n \geq 0}: \quad a_n \leq a_{n+1} \quad \forall n \in \mathbb{N}$
- DECRESCENTE $\quad a_n \geq a_{n+1} \quad \forall n \in \mathbb{N}$

$$a_n = n^3 + 4\sqrt{n} - 2$$

MON CRESCENTE $\iff a_{n+1} - a_n \geq 0 \quad \forall n \in \mathbb{N}$

$$\iff (n+1)^3 + 4\sqrt{n+1} - 2 - (n^3 + 4\sqrt{n} - 2) \geq 0$$

$$\iff \cancel{n^3} + 3n^2 + 3n + 1 + 4\sqrt{n+1} \cancel{-2} - \cancel{n^3} - 4\sqrt{n} \cancel{+2} \geq 0$$

$$\iff \underbrace{3n^2 + 3n + 1}_{\substack{\geq 0 \\ (>0)}} + 4\underbrace{(\sqrt{n+1} - \sqrt{n})}_{\geq 0 \ (>0)} \geq 0$$

MOTIVO? LA SUCCESSIONE $b_n = \sqrt{n}$ È MONOTONA CRESCENTE

QUINDI a_n È MONOTONA CRESCENTE. $\square$

$f(x) = \sqrt{x}$

b) NON MONOTONA

c) ?