Algebra Lineare 14/12/16

$\boxed{28}$ 1

$$A_k = \begin{pmatrix} 2(2-5k) & 2(1-3k) & k+1 \\ 6(3k-1) & 11k-3 & k \\ 0 & 0 & 1+k^2 \end{pmatrix}.$$

A $\left(\begin{array}{l}\text{Trovare } k_1 < k_2 \text{ t.c.} \\ \text{per } k \neq k_1, k_2 \text{ il sist.} \\ A_k \cdot x = b \text{ ha soluz.} \\ \text{unica } \forall b \in \mathbb{R}^3\end{array}\right) \iff \left(\begin{array}{c}\text{Trovare } k_1, k_2 \\ \text{t.c.} \\ \det(A_k) \neq 0 \\ \text{per } k \neq k_1, k_2\end{array}\right)$

cioè : trovare le soluz $K_{1,2}$ dell'equazione

$$\det(A_k) = 0.$$

$$\begin{pmatrix} 2(2-5k) & 2(1-3k) & k+1 \\ 6(3k-1) & 11k-3 & k \\ 0 & 0 & 1+k^2 \end{pmatrix}.$$

Devo risolvere $\det \begin{pmatrix} 2-5k & 1-3k \\ 18k-6 & 11k-3 \end{pmatrix} = 0$

calcoli...

$\boxed{28}$ (2) $E: \; 2x - 3y + 5z = 4$

$$F_k = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} + \operatorname{Span} \begin{pmatrix} 3 \\ 5k+2 \\ 3k \end{pmatrix} \begin{pmatrix} 3k+2 \\ 2k+3 \\ 1 \end{pmatrix}$$

(A) dim E + eq. par. $\quad \dim(E) = 2$

$$E = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \operatorname{Span} \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}$$

(B) dim F_k + eq. cart. per $k = 2$

dim $F_k = 2$ a meno che siano lin. dip.

$$\begin{pmatrix} 3 \\ 5k+2 \\ 3k \end{pmatrix} \quad \begin{pmatrix} 3k+2 \\ 2k+3 \\ 1 \end{pmatrix}$$

$$\det\left(\quad\right) = 3 - 9k^2 - 6k$$
$$= -3(3k^2+2k-1)$$
$$= -3(3k-1)(k+1)$$

Per i valori $k=-1$ e $k=1/3$ che annullano questo det. sostituisco e vedo se sono davvero lin. dip.:

$k=-1$ $\quad \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ <u>sì</u> $\qquad k=1/3$ $\quad \begin{pmatrix} 3 \\ 11/3 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 11/3 \\ 1 \end{pmatrix}$ <u>sì</u>

Conclusione: $\dim F_k = 1$ per $k=-1$, $k=1/3$; $\dim F_k = 2$ altrimenti.

(c) Provare che $E \parallel F_k$ e trovare quando coincidono.

$E: \ 2x - 3y + 5z = 4$

$F_k = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} + \text{Span} \begin{pmatrix} 3 \\ 5k+2 \\ 3k \end{pmatrix} \begin{pmatrix} 3k+2 \\ 2k+3 \\ 1 \end{pmatrix}$

$E // F_k$ significa (giacitura F_k) $\subset$ (giacitura di E)

ovvero: $\begin{pmatrix} 3 \\ 5k+2 \\ 3k \end{pmatrix} \begin{pmatrix} 3k+2 \\ 2k+3 \\ 1 \end{pmatrix}$ soddisfano $2x - 3y + 5z = 0$

infatti: $6 - 15k - 6 + 15k = 0$ ✓

$6k + 4 - 6k - 9 + 5 = 0$ ✓

Coincidono se $\begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} \in E$ cioè $2 - 3k + 10 = 4$

$k = 8/3$.

29 ① Per $k \in \mathbb{R}$ consideriamo le $f_k : \mathbb{R}^3 \to \mathbb{R}^3$ lin. t.c.

$$f_k\begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}-k\\0\\k+2\end{pmatrix}$$

$$f_k\begin{pmatrix}-1\\0\\k\end{pmatrix} = \begin{pmatrix}4-k\\2\\1-2k\end{pmatrix}$$

$$f_k\begin{pmatrix}0\\k\\1\end{pmatrix} = \begin{pmatrix}0\\k\\1\end{pmatrix}$$

Ⓐ Quante sono?

Voglio le $f_k : \mathbb{R}^3 \to \mathbb{R}^3$ t.c.

$$f_k(v_k^{(j)}) = w_k^{(j)} \qquad j = 1, 2, 3$$

$\left(v_k^{(1)}, v_k^{(2)}, v_k^{(3)} \right)$ sono base?

→ Sì: f_k esiste unica

→ No

Osservando che $v_k^{(1)}, v_k^{(2)}$ sono sempre lin indip. concludo che esistono α, β t.c. $v_k^{(3)} = \alpha \cdot v_k^{(1)} + \beta \cdot v_k^{(2)}$

Mi chiedo se

$$\alpha \cdot w_k^{(1)} + \beta \cdot w_k^{(2)} \overset{?}{=} w_k^{(3)}$$

No

Non esiste alcuna f_k

Sì

la condizione $f_k(v_k^{(3)}) = w_k^{(3)}$ è conseguenza

$$\circledast \begin{cases} f_k(v_k^{(1)}) = w_k^{(1)} \\ f_k(v_k^{(2)}) = w_k^{(2)} \end{cases}$$

Dunque alla f_k si chiedono solo le $\circledast$; se si completa $v_k^{(1)}, v_k^{(2)}$ a base di $\mathbb{R}^3$

posso aggiungere alla ⊗

$f_k(u) = y \quad y \in \mathbb{R}^3$ qualsiasi

$\Longrightarrow$ esistono infinite f_k.

Per noi:

$$\det\begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & k \\ -1 & k & 1 \end{pmatrix} = k + 2 - k^2$$

$$= -(k^2 - k - 2) = -(k-2)(k+1)$$

Per $k \neq 2, k \neq -1$ esiste un'unica f_k.

$$k = -1 \qquad \left(v^{(1)}, v^{(2)}, v^{(3)}\right) = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$v^{(3)} = -\frac{1}{2}\left(v^{(1)} + v^{(2)}\right)$$

$$\left(w^{(1)}, w^{(2)}, w^{(3)}\right) = \begin{pmatrix} 1 & 5 & 0 \\ 0 & 2 & -1 \\ -1 & 3 & 1 \end{pmatrix}$$

$$w^{(3)} \neq -\frac{1}{2}\left(w^{(1)} + w^{(2)}\right)$$ **No**

non esiste alcuna f_k.

$k=2$ $\quad (v^{(1)}, v^{(2)}, v^{(3)}) = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ -1 & 2 & 1 \end{pmatrix}$ $\quad v^{(3)} = v^{(1)} + v^{(2)}$

$$(w^{(1)}, w^{(2)}, w^{(3)}) = \begin{pmatrix} -2 & 2 & 0 \\ 0 & 2 & 2 \\ 4 & -3 & 1 \end{pmatrix}$$

$$w^{(3)} \overset{?}{=} w^{(1)} + w^{(2)}$$

Sì

esistono infinite f_k.

(B) Per $k=1$ trovare $[f_1]_{\mathcal{E}^{(3)}}^{\mathcal{E}^{(3)}}$.

Se chiamo $A = [f_1]_{\mathcal{E}^{(3)}}^{\mathcal{E}^{(3)}}$ ho $f_1(x) = A \cdot x$ $\forall x \in \mathbb{R}^3$

Conosco

$$f_1(v_1^{(j)}) = w_1^{(j)} \qquad j = 1, 2, 3$$

quindi A deve soddisfare

$$A \cdot v_1^{(j)} = w_1^{(j)} \qquad j = 1, 2, 3$$

$$A \cdot (v_1^{(1)}, v_1^{(2)}, v_1^{(3)}) = (w_1^{(1)}, w_1^{(2)}, w_1^{(3)})$$

$$\Rightarrow A = (w_1^{(1)}, w_1^{(2)}, w_1^{(3)}) \cdot (v_1^{(1)}, v_1^{(2)}, v_1^{(3)})^{-1}$$

calcoli...

$\boxed{30}$ (1) $E : \begin{cases} x_1 - 2x_2 + x_3 + x_4 = 1 \\ 2x_1 + x_2 + 3x_3 - x_4 = -1 \end{cases}$

(A) Eq. param. pr E :

$$E = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \text{Span} \begin{pmatrix} +7 \\ +1 \\ -5 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ +4 \\ +1 \\ +7 \end{pmatrix}$$

$$\cancel{\begin{cases} -2x_2 + x_4 = 1 \\ x_2 - x_4 = -1 \end{cases}}$$

$$F_k = \begin{pmatrix} 1-k \\ 1 \\ k \\ 2 \end{pmatrix} + \text{Span} \begin{pmatrix} -k \\ 2k^2-3k+2 \\ 1 \\ 2k^2-5k+5 \end{pmatrix} \begin{pmatrix} 2(1-k) \\ 2k(k-1) \\ k-1 \\ 2k^2-3k+1 \end{pmatrix}$$

(B) Trovare eq. cart. di F_k per $k=0$

$$\begin{pmatrix}1\\1\\0\\2\end{pmatrix} + \mathrm{Span}\left(\begin{pmatrix}0\\2\\1\\5\end{pmatrix}\begin{pmatrix}2\\0\\-1\\1\end{pmatrix}\right)$$

$$\begin{cases} \cancel{-2}x + \overset{-1}{\cancel{(+2)}}\,y + \overset{2}{\cancel{-4}}\,z + 0\cdot w = 0 \\ 0\cdot x + \underset{-3}{\cancel{6}}\,y + \underset{1}{\cancel{(-2)}}\,z + \underset{1}{\cancel{(-2)}}\,w = -1 \end{cases}$$

(C) $\dim F_k = ?$

$\dim F_k = 2$ a meno che

$$\begin{pmatrix} -k \\ 2k^2-3k+2 \\ 1 \\ 2k^2-5k+5 \end{pmatrix} \begin{pmatrix} 2(1-k) \\ 2k(k-1) \\ k-1 \\ 2k^2-3k+1 \end{pmatrix}$$

non siano lin. dip.

$$-k^2+k-2+2k = -(k^2-3k+2)$$
$$= -(k-1)(k-2)$$

$k=1$ $\begin{pmatrix} -1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ SI $\Rightarrow \dim = 1$

$k=2$ $\begin{pmatrix} -2 \\ 4 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \\ 1 \\ 3 \end{pmatrix}$ SI $\Rightarrow \dim = 1$

$\boxed{30}$ ② $A = \begin{pmatrix} 0 & 1 & 2 \\ -2 & -3 & 2 \\ -1 & 1 & 3 \end{pmatrix}$ $f = f_A$

$v_1 = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ $v_3 = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}$

Ⓐ Provare che $(v_1, v_2, v_3) = \mathcal{B}$ è base

$$\det \begin{pmatrix} 5 & 3 & -2 \\ 2 & 1 & -3 \\ 1 & 1 & 5 \end{pmatrix} = 25 - 9 + 2 - 4 + 15 - 30 = -1$$

SÌ

Ⓑ Calcolare A^{-1}:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ -2 & -3 & 2 \\ -1 & 1 & 3 \end{pmatrix}$$

$$\det(A) = -2 - 4 - 6 + 6 = -6$$

$$A^{-1} = -\frac{1}{6} \begin{pmatrix} -11 & -1 & \dots \\ - \dots & \dots & - \dots \\ \dots & - \dots & \dots \end{pmatrix}$$

ⓒ Trovare $[f]^{\mathcal{E}^{(3)}}_{\mathcal{B}}$, $[f]^{\mathcal{B}}_{\mathcal{E}^{(3)}}$, $[f]^{\mathcal{B}}_{\mathcal{B}}$.

$f = A$

$[f]_{\mathcal{B}}^{\mathcal{E}^{(3)}}$:

$$\underset{\substack{\| \\ A}}{f} \cdot \mathcal{B} = \underset{\substack{\| \\ I_3}}{\mathcal{E}^{(3)}} \cdot [f]_{\mathcal{B}}^{\mathcal{E}^{(3)}}$$

$$\Longrightarrow [f]_{\mathcal{B}}^{\mathcal{E}^{(3)}} = A \cdot \mathcal{B}$$

$[f]_{\mathcal{E}^{(3)}}^{\mathcal{B}}$:

$$\underset{\substack{\| \\ A}}{f} \cdot \underset{\substack{\| \\ I_3}}{\mathcal{E}^{(3)}} = \mathcal{B} \cdot [f]_{\mathcal{E}^{(3)}}^{\mathcal{B}}$$

$$\Longrightarrow [f]_{\mathcal{E}^{(3)}}^{\mathcal{B}} = \mathcal{B}^{-1} \cdot A$$

$[f]_{\mathcal{B}}^{\mathcal{B}}$:

$$f \cdot \mathcal{B} = \mathcal{B} \cdot [f]_{\mathcal{B}}^{\mathcal{B}}$$

$$\Longrightarrow [f]_{\mathcal{B}}^{\mathcal{B}} = \mathcal{B}^{-1} \cdot A \cdot \mathcal{B} .$$