Geometria 8/5/14

33. $5x^2-6xy+5y^2-10x+6y-k=0$

$$\begin{pmatrix} 5 & -3 & -5 \\ -3 & 5 & 3 \\ -5 & 3 & -k \end{pmatrix}$$

$d_1>0 \quad d_2>0$

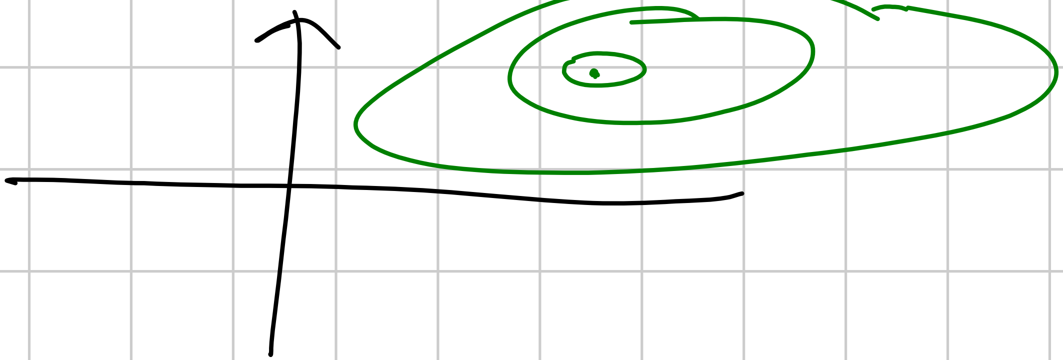
$d_3 = -25k+90-125+9k-45$

$= -16k-80 = -16(k+5)$

$k>-5$ ellisse

$k<-5$ $\emptyset$

$k=-5 \quad ++0$ un punto $X^2+Y^2=0$ _



35. $kx^2+4xy+y^2-4x-2y+5=0$

$$\begin{pmatrix} k & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 5 \end{pmatrix}$$

$d_1>0$ (scambiando x/y)

$d_2 = k-4$

$$d_3 = \det\begin{pmatrix} k-4 & 2 & -2 \\ 0 & 1 & -1 \\ 0 & -1 & 5 \end{pmatrix} = 4(k-4)$$

$k>4$ $\emptyset$

$k<4$ iperbole

$k=4$ $4x^2+4xy+y^2-4x-2y+5=0$

$(2x+y)^2-2(2x+y)+\underset{1+4}{\underset{\shortparallel}{5}}=0$

$(2x+y-1)^2+4=0 \Longrightarrow \emptyset$

36. $x^2-2xy+2ky^2+2kx+2y+1=0$

$$\begin{pmatrix} 1 & -1 & k \\ -1 & 2k & 1 \\ k & 1 & 1 \end{pmatrix}$$

$d_1>0$ $d_2=2k-1$

$d_3=2k-k-k-2k^3-1-1$

$=-2(k^3+1)$

$k>\frac{1}{2}$ $d_2>0$, $d_3<0$ $\Longrightarrow$ ellisse

$k = 1/2 \quad d_2 = 0 , \; d_3 \neq 0 \implies$ parabola

$k < 1/2 , \; k \neq -1 \implies$ iperbole

$k = -1 : \quad x^2 - 2xy - 2y^2 - 2x + 2y + 1 = 0$

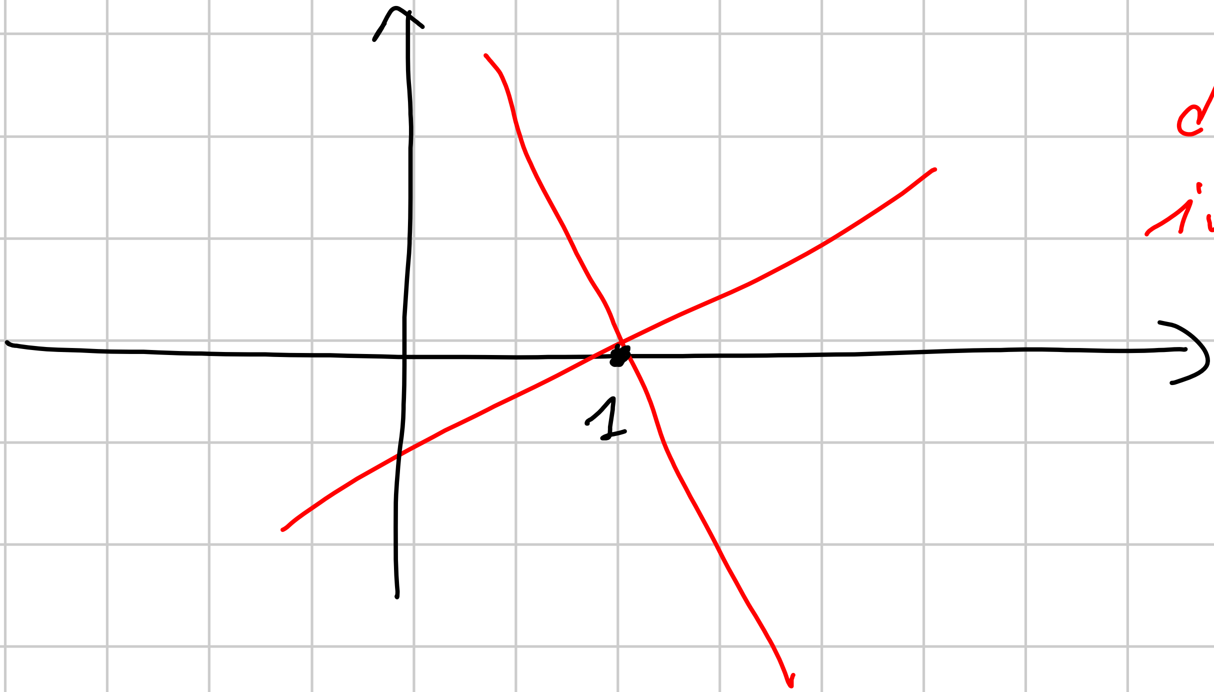
$d_2 \neq 0 \quad d_3 = 0 \implies$ l'equaz. si fattorizza

$$\left(x + a \cdot y + b\right)\left(x - \frac{2}{a} \cdot y + \frac{1}{b}\right) = 0$$

$$\begin{cases} a - \dfrac{2}{a} = -2 \\ b + \dfrac{1}{b} = -2 \implies b^2 + 2b + 1 = 0 \implies b = -1 \\ \dfrac{a}{b} - 2\dfrac{b}{a} = 2 \quad \checkmark \end{cases}$$

$$a^2+2a-2=0 \qquad a=-1\pm\sqrt{3} \qquad -\frac{2}{a}=-1\mp\sqrt{3}$$

$$\left(x+(-1+\sqrt{3})y-1\right)\left(x+(-1-\sqrt{3})y-1\right)=0$$



due rette incidenti

37. $(k+1)x^2 + (k-1)y^2 + 2kx + 2y - 1 = 0$

$$\begin{pmatrix} k+1 & 0 & k \\ 0 & k-1 & 1 \\ k & 1 & -1 \end{pmatrix}$$

$d_2 = k^2 - 1$

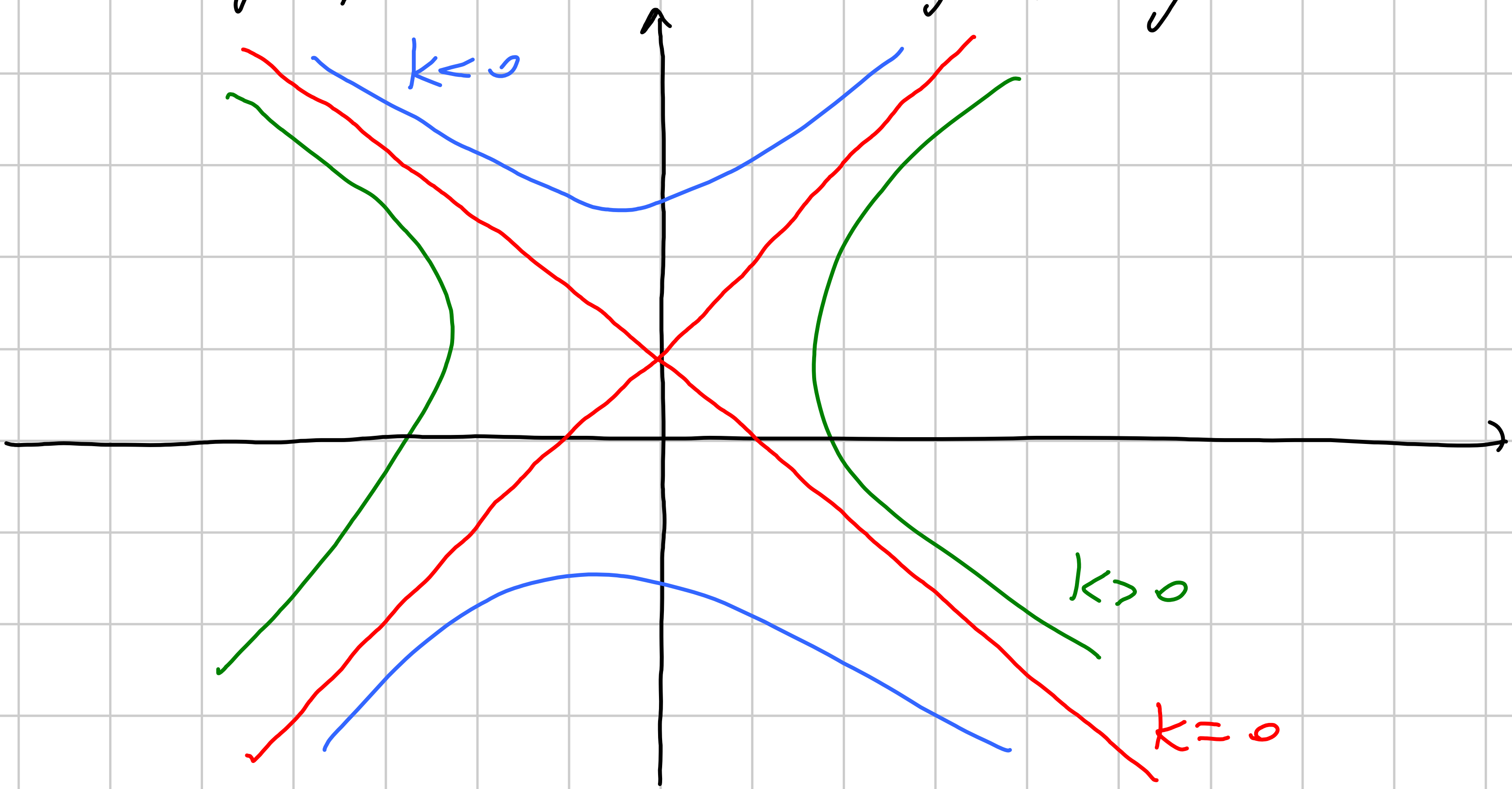
(autoval di Q $\quad k \pm 1$)

$$d_3 = \cancel{-k^2} \cancel{+1} - k^3 \cancel{+k^2} - k \cancel{-1}$$

$$= -k(k^2+1)$$

k	-1		0		1	
$d_1<0$, $d_2>0$, $d_3>0$, $- - +$ ellisse	parab.	iperb	degenere	iperb	parab.	$d_1>0$, $d_2>0$, $d_3<0$, $+ + -$ ellisse

$$x^2 - y^2 + 2y - 1 = 0 \qquad x^2 = (y-1)^2 \qquad y = 1 \pm x$$



38. $kx^2 + 2\sqrt{k}xy + 3y^2 + 2\sqrt{k}x + k = 0$ $\quad k \geq 0$

$$\begin{pmatrix} k & \sqrt{k} & \sqrt{k} \\ \sqrt{k} & 3 & 0 \\ \sqrt{k} & 0 & k \end{pmatrix}$$

$d_1 > 0 \quad d_2 = 2k \geq 0$

$d_3 = 3k^2 - 3k - k^2$
$= k(2k-3)$

$k = 0$ degenere $\quad y = 0$ una volta (doppia)

$0 < k < 3/2$ ellisse

$k = 3/2$ un punto (autoval + + 0 ...)

$k > 3/2$ $\emptyset$

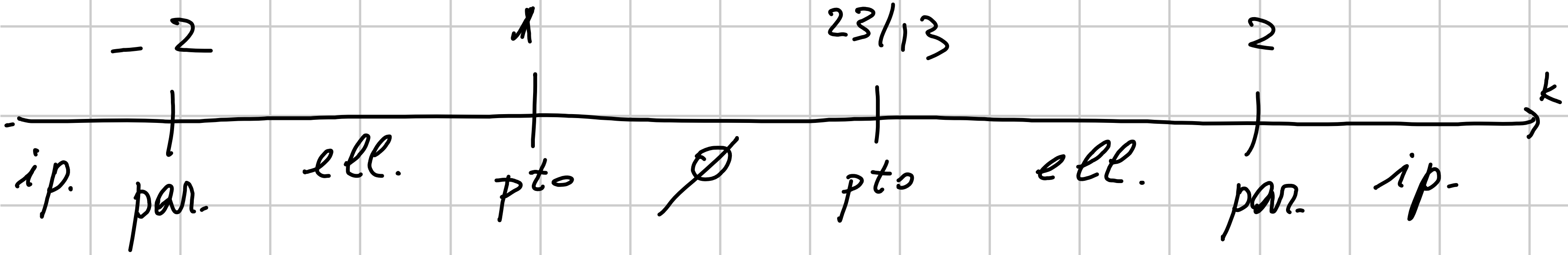
39. $x^2 - 2kxy + 4y^2 - 4x + 6y + \frac{13}{3} = 0$

$$\begin{pmatrix} 1 & -k & -2 \\ -k & 4 & 3 \\ -2 & 3 & 13/3 \end{pmatrix}$$

$d_1 > 0 \quad d_2 = 4 - k^2$

$d_3 = \ldots = -\frac{1}{3}(13k^2 - 36k + 23)$

$= -\frac{1}{3}(k-1)(13k-23)$



(2) $8x^2-12xy+17y^2+4x+12y+4=0$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 17 & 6 \\ 2 & 6 & 4 \end{pmatrix}$$

$d_1>0 \quad d_2>0$

$$d_3 = 2\cdot \det\begin{pmatrix} 8 & -6 & 1 \\ -30 & 35 & 0 \\ -14 & 18 & 0 \end{pmatrix} < 0$$

$\Longrightarrow$ ellisse

Trovare i cambi di coordinate.

- affine che la trasforma in $X^2+Y^2=1$
- isometrico " " " in $\dfrac{w^2}{a^2}+\dfrac{u^2}{b^2}=1$

$$8x^2-12xy+17y^2+4x+12y+4=0$$

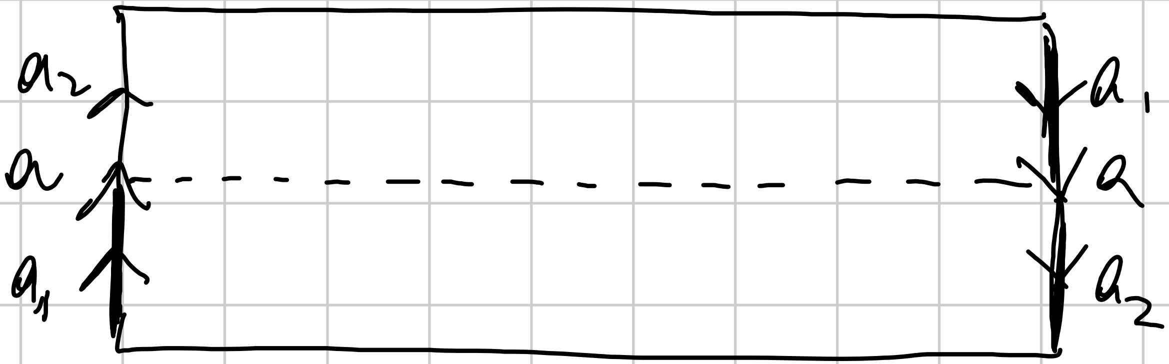
$$(2x+y+2)^2+(2x-4y-1)^2=1$$

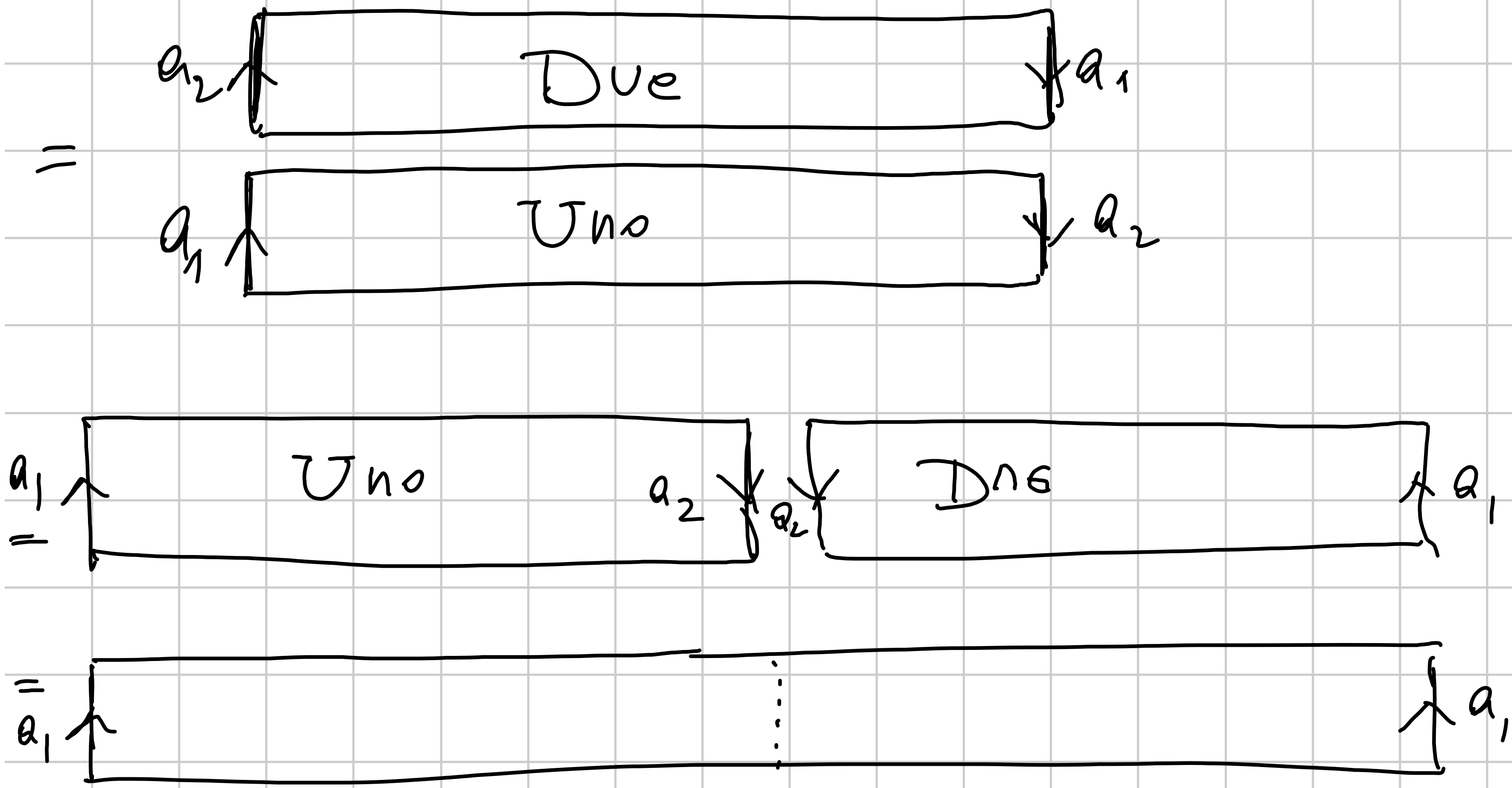
$$\begin{cases} X = 2x+y+2 \\ Y = 2x-4y-1 \end{cases} \qquad \begin{cases} z = \dfrac{2x+y+2}{\sqrt{5}} \\[2ex] u = \dfrac{x-2y-1/2}{\sqrt{5}} \end{cases}$$

$$\begin{pmatrix} z \\ u \end{pmatrix} = \underbrace{\frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}}_{\text{ortogonale (riflessione)}} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2/\sqrt{5} \\ -1/2\sqrt{5} \end{pmatrix}$$

$$\frac{z^2}{1/5} + \frac{u^2}{1/20} = 1 \qquad a = \frac{1}{\sqrt{5}} \quad b = \frac{1}{2\sqrt{5}}.$$

Secondo foglio del 18/4/14
(era sbagliata la prima figura).




=
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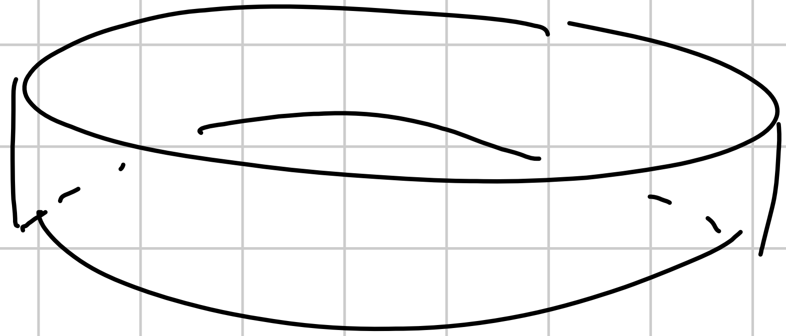
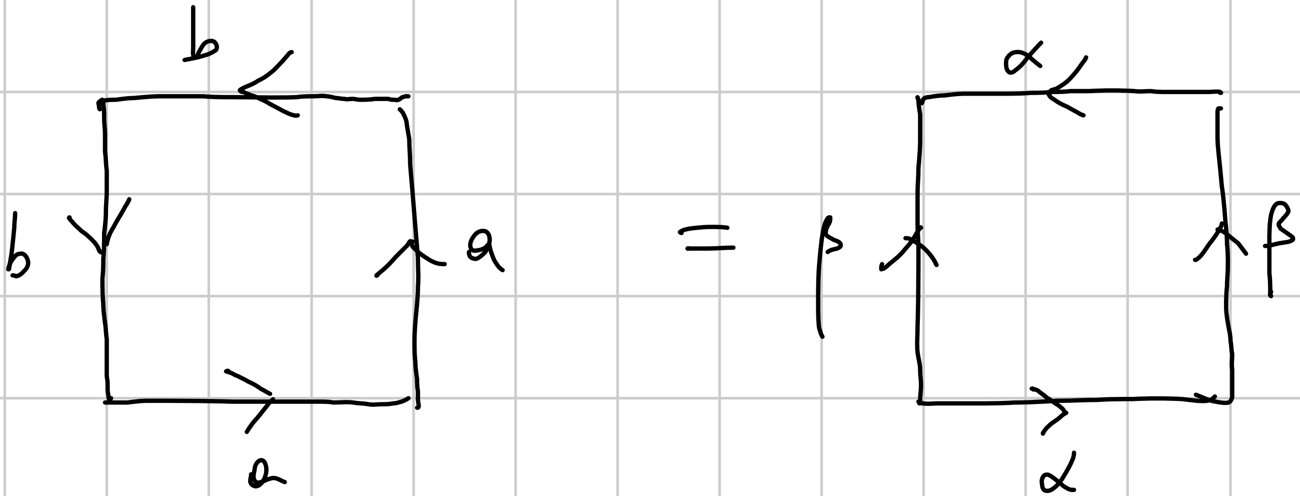


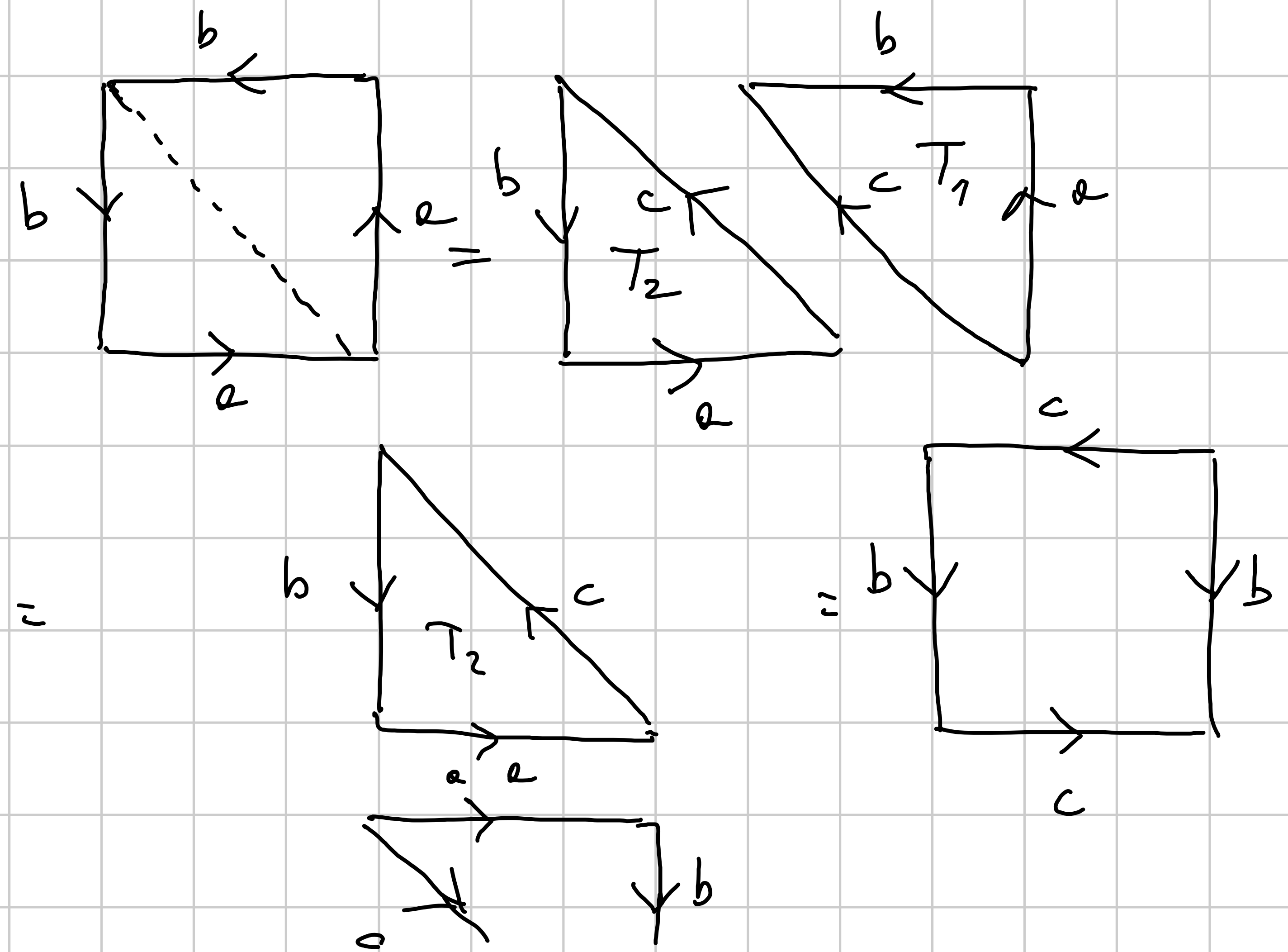
Intrinsecamente ottengo un cilindro, ma se prima realizzo il Möbius e poi taglio ottengo un oggetto che è intrinsecamente un cilindro ma in $\mathbb{R}^3$ non viene

estrinsecamente

(2)



(potete provare con un foglio o un pezzo di camera d'aria: non ce la farete mai)


b
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a
a
=
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c
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b
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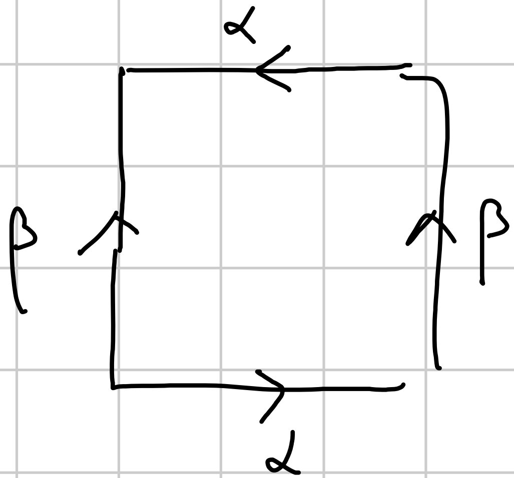
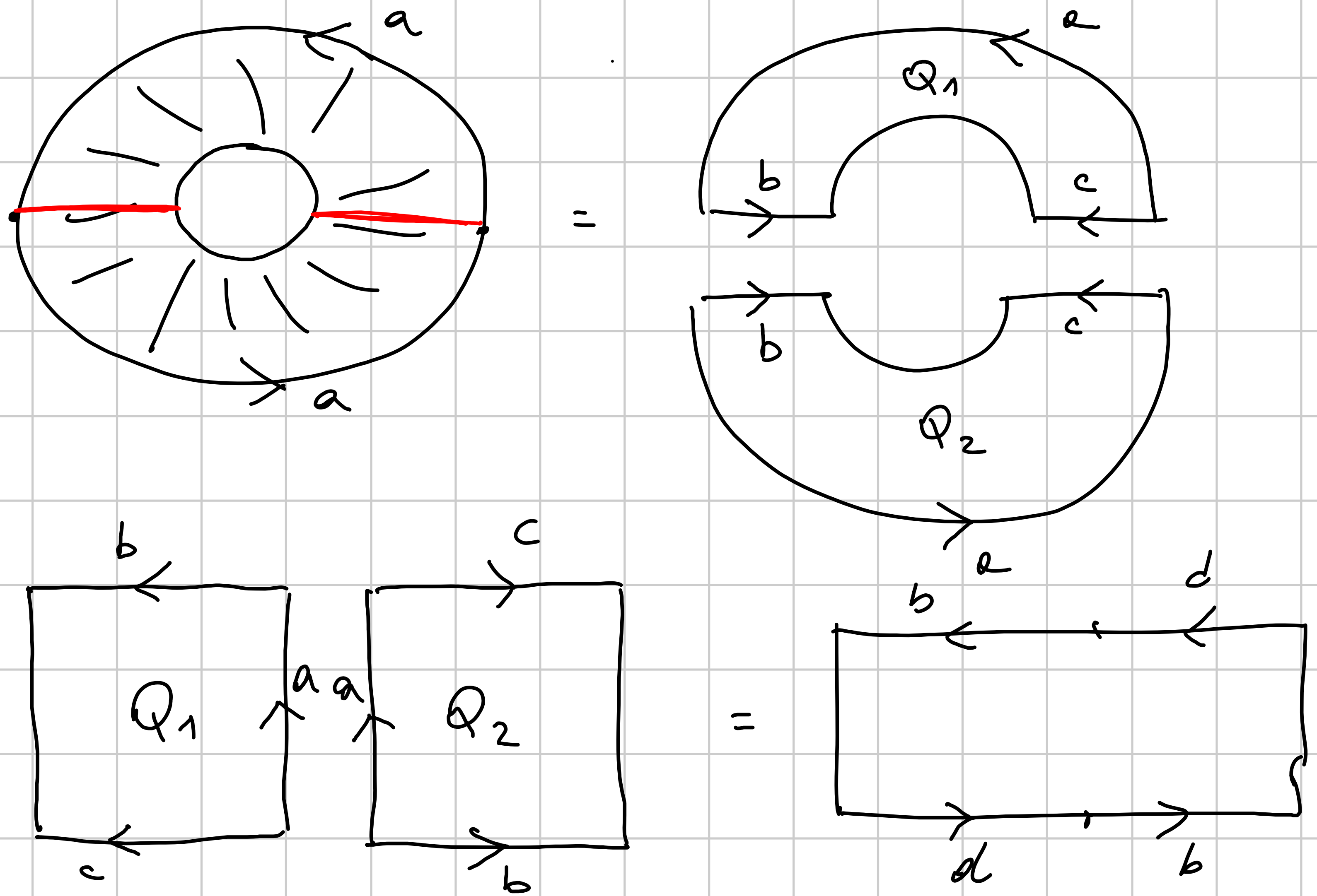
c

$=$ 

<u>Esercizio</u>: togliendo un dischetto a un $\mathbb{P}^2(\mathbb{R})$ si trova un nastro di Möbius.


a
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=
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Q2
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b
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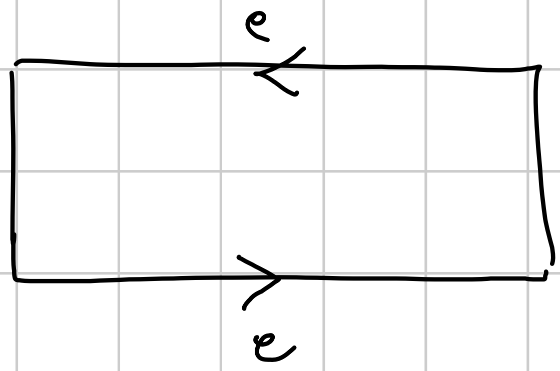
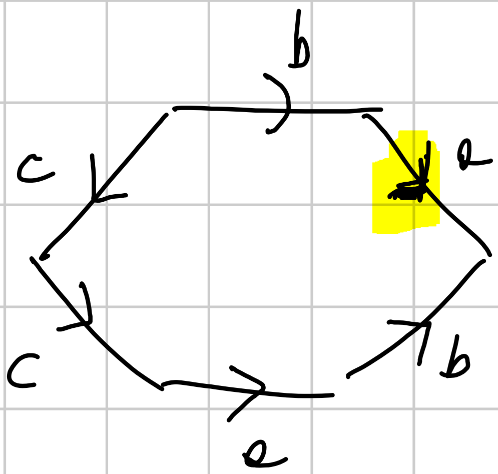
b

=



= Möbius

3



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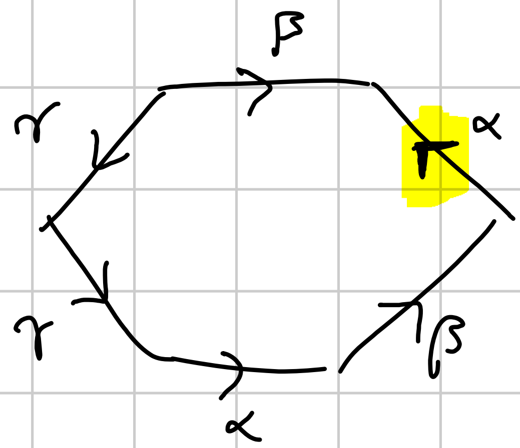

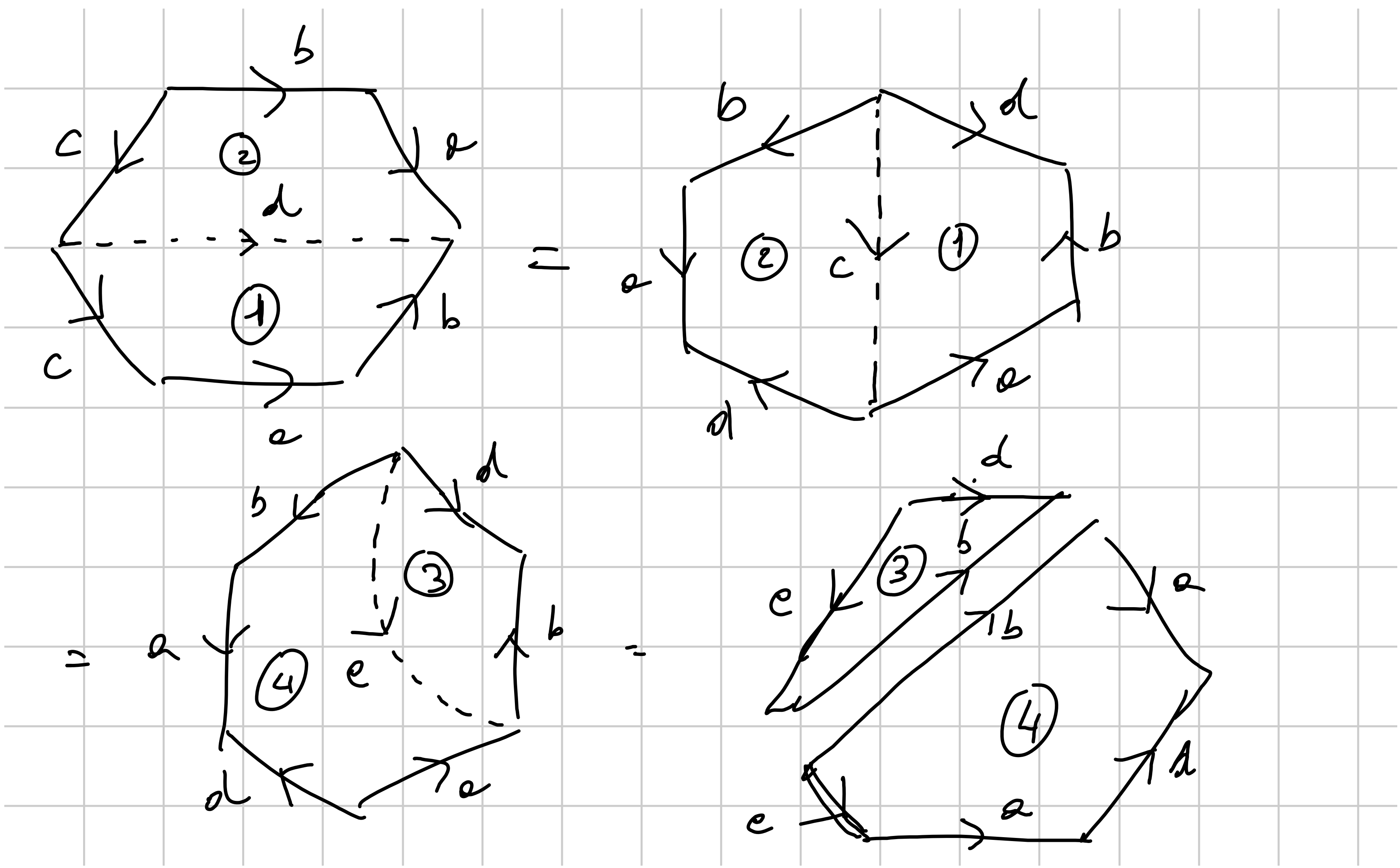

b
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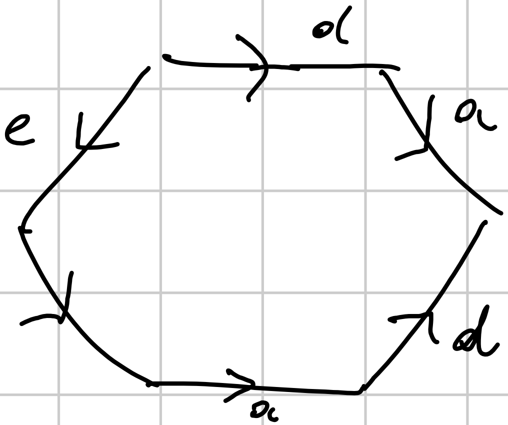
= 
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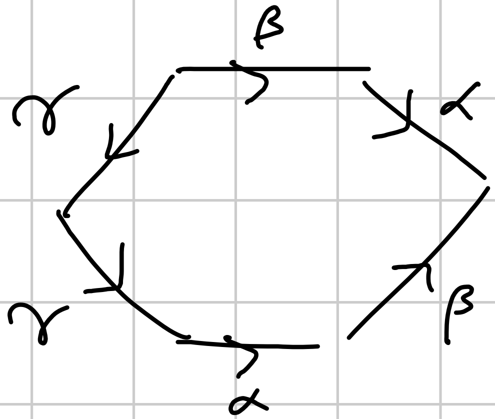
= 
β
γ
α
γ
β
α

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