Algebra Lineare – Esercizi del 12/11/09

(1) Verificare che $V = W \oplus Z$ ed esibire le proiezioni associate a tale decomposizione:

(a) $V = \mathbb{R}^2 \quad W = \mathrm{Span}\begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad Z = \{x : 5x_1 - 4x_2 = 0\}$

(b) $V = \mathbb{R}^3 \quad W = \mathrm{Span}\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad Z = \mathrm{Span}\left(\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}\right)$

(c) $V = \mathbb{R}^3 \quad W = \mathrm{Span}\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} \quad Z = \{x : x_1 + x_2 + x_3 = 0\}$

(d) $V = \mathbb{R}^3 \quad W = \left\{x : \begin{array}{l} x_1 - x_2 + 2x_3 = 0 \\ 2x_1 + x_2 - x_3 = 0 \end{array}\right\} \quad Z = \mathrm{Span}\left(\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}\right)$

(e) $V = \mathbb{R}^3 \quad W = \left\{x : \begin{array}{l} 2x_1 - 3x_2 = 0 \\ 4x_2 + 5x_3 = 0 \end{array}\right\} \quad Z = \{x : 6x_1 - 7x_2 + 2x_3 = 0\}$

(f) $V = \mathcal{M}_{2\times 2}(\mathbb{R}) \quad W = \mathrm{Span}\left(\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}\right)$

$Z = \mathrm{Span}\left(\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 1 & 0 \end{pmatrix}\right)$

(g) $V = \mathbb{R}_{\leq 3}[t] \quad W = \{p(t) : p'(-1) = 0\}$

$Z = \mathrm{Span}(1+t,\ t+t^2,\ t-2t^3)$

(h) $V = \mathcal{F}(\{a,b,c\}, \mathbb{R}) \quad W = \{x : 3x(a) - 2x(b) + x(c) = 0\}$

$Z = \mathrm{Span}\left(\begin{cases} a \mapsto -1 \\ b \mapsto 5 \\ c \mapsto 0 \end{cases}, \begin{cases} a \mapsto 2 \\ b \mapsto 0 \\ c \mapsto 3 \end{cases}\right)$

(2) Eseguire quelli che sono possibili tra i prodotti righe x colonne $A \cdot B$ e $B \cdot A$, e quando lo sono entrambi confrontarli

(a) $A = \begin{pmatrix} 1 & 3 & -1/2 \\ 4 & 0 & 2 \end{pmatrix}$ $\quad B = \begin{pmatrix} 5 & 1 \\ -1 & 3 \end{pmatrix}$

(b) $A = \begin{pmatrix} -1 & 2 & 1 \\ 4 & -3 & 5 \end{pmatrix}$ $\quad B = \begin{pmatrix} 6 & 1 \\ -1 & 3 \\ -5 & 4 \end{pmatrix}$

(c) $A = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$ $\quad B = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$

(d) $A = \begin{pmatrix} -1 & 4 \\ 2 & 3 \\ 2 & -3 \\ 0 & 5 \end{pmatrix}$ $\quad B = \begin{pmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 0 & 4 \\ 0 & 1 & -2 & 0 \end{pmatrix}$

(3) Esibire basi di nucleo e immagine della applicazione lineare associata alla matrice data:

(a) $\begin{pmatrix} -1 & 2 & 4 \\ 3 & 1 & -5 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 0 \\ +2 & 5 & -1 \\ 3 & 1 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 4 & -2 & 1 \\ 2 & 1 & 5 & 0 \\ -1 & 3 & 1 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 & 1 \\ -2 & -1 & -4 \\ 3 & 1 & 7 \\ -2 & -3 & 0 \end{pmatrix}$