"Geometria" — Esercizi del 13/3/09

(1) Determinare la distanza in $\mathbb{R}^n$, dotato del prodotto scalare canonico, del vettore v dal sottospazio X:

(a) $n=2$, $v=\begin{pmatrix}3\\-2\end{pmatrix}$, $X=\{x : 7x_1=24x_2\}$

(b) $n=3$, $v=\begin{pmatrix}2\\-1\\3\end{pmatrix}$, $X=\{x : x_1+3x_2+2x_3=0\}$

(c) $n=3$, $v=\begin{pmatrix}-2\\4\\-3\end{pmatrix}$, $X=\mathrm{Span}\left(\begin{pmatrix}1\\-2\\4\end{pmatrix},\begin{pmatrix}3\\1\\-1\end{pmatrix}\right)$

(d) $n=4$, $v=\begin{pmatrix}1\\2\\-1\\1\end{pmatrix}$, $X=\{x : 2x_1-x_2+x_3+x_4=0\}$

(e) $n=4$, $v=\begin{pmatrix}-1\\1\\2\\1\end{pmatrix}$, $X=\left\{x : \begin{array}{l}x_1-x_2+2x_3=0\\ 2x_2-3x_3+x_4=0\end{array}\right\}$

(f) $n=4$, $v=\begin{pmatrix}3\\1\\-2\\5\end{pmatrix}$, $X=\mathrm{Span}\left(\begin{pmatrix}2\\-1\\3\\1\end{pmatrix}\right)$

(2) Ortonormalizzare rispetto al prodotto scalare canonico di $\mathbb{C}^n$ le seguenti basi di $V\subset\mathbb{C}^n$:

(a) $V=\mathrm{Span}\left(\begin{pmatrix}2+i\\3-2i\end{pmatrix}\right)\subset\mathbb{C}^2$, $\mathcal{B}=\left(\begin{pmatrix}2+i\\3-2i\end{pmatrix}\right)$

(b) $V = \{ z \in \mathbb{C}^3 : 2z_1 - iz_2 + z_3 = 0 \}$

$$\mathcal{B} = \left(\begin{pmatrix} 1 \\ i \\ -3 \end{pmatrix}, \begin{pmatrix} i \\ 2+i \\ -1 \end{pmatrix} \right)$$

(c) $V = \mathbb{C}^2$, $\mathcal{B} = \left(\begin{pmatrix} 2-3i \\ 4+i \end{pmatrix}, \begin{pmatrix} -7i \\ 1+i \end{pmatrix} \right)$

(d) $V = \mathrm{Span}(\mathcal{B}) \subset \mathbb{C}^4$

$$\mathcal{B} = \left(\begin{pmatrix} i \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -i \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2i \\ 1 \\ 1 \\ 0 \end{pmatrix} \right)$$

(3) Stabilire se le seguenti matrici sono hermitiane e in tal caso se sono definite positive:

(a) $\begin{pmatrix} 2i & 1 \\ 1 & 3 \end{pmatrix}$, $\begin{pmatrix} 2 & 1+i \\ 1-i & -1 \end{pmatrix}$, $\begin{pmatrix} 3 & 2-i \\ 2+i & 5 \end{pmatrix}$,

$\begin{pmatrix} 1 & 2-5i \\ 2+5i & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & i & 1-i \\ -i & -1 & 0 \\ 1+i & 0 & 2 \end{pmatrix}$, $\begin{pmatrix} 2 & 0 & 7+i \\ 0 & 0 & 1+2i \\ 7-i & 1-2i & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 3-i & 1+i \\ 3+i & 3-2i & 0 \\ 1-i & 0 & 4 \end{pmatrix}$, $\begin{pmatrix} 2 & 1+i & 0 \\ 1-i & 5 & 2-i \\ 0 & 2+i & 3 \end{pmatrix}$