Algebra Lineare – Esercizi del 23/10/08

Esibire basi dei seguenti spazi vettoriali:

(1) $\{x \in \mathbb{R}^3 : 2x_1 - 5x_2 + 7x_3 = 0\}$

(2) $\{x \in \mathbb{R}^4 : \begin{array}{l} 3x_1 - 2x_2 + 4x_3 + 5x_4 = 0 \\ -x_1 + 4x_2 - 7x_3 + 3x_4 = 0 \\ 7x_1 - 8x_2 + 15x_3 + 7x_4 = 0 \end{array}\}$

(3) $\{x \in \mathbb{R}^n : \sum_{j=1}^{n} (-j)^{j+1} x_j = 0\}$

(4) $\{x \in \mathbb{R}^n : \sum_{j=1}^{n} (-1)^j x_j = \sum_{j=1}^{n} \left(1 + (-1)^{j+1}\right) x_j = 0\}$

(5) $\{A \in \mathcal{M}_{2\times 3}(\mathbb{R}) : \begin{array}{l} a_{11} + a_{22} + a_{31} = 0 \\ a_{12} + a_{21} + a_{32} = 0 \end{array}\}$

(6) $\{A \in \mathcal{M}_{3\times 2}(\mathbb{R}) : \begin{array}{l} a_{11} + 2a_{12} + 3a_{31} = 0 \\ a_{11} - 2a_{22} + 4a_{32} = 0 \\ -a_{22} + a_{21} + 2a_{32} = 0 \\ 2a_{12} + 2a_{21} + 3a_{31} = 0 \end{array}\}$

(7) $\mathcal{S}_n(\mathbb{R})$, $\mathcal{A}_n(\mathbb{R})$

(8) $\{A \in \mathcal{S}_n(\mathbb{R}) : \mathrm{tr}(A) = 0\}$

(9) $X = \mathcal{F}(\{a,b,c\}, \mathbb{R})$

(10) $\{f \in X : 3f(a) - 2f(b) + 5f(c) = 0\}$

(11) $\{f \in X : \begin{array}{l} 2f(a) - 3f(b) - 4f(c) = 0 \\ 4f(a) + 5f(b) - 7f(c) = 0 \end{array}\}$

(12) $\{p(x) \in \mathbb{R}[x] : \deg(p(x)) \leq d,\ p(1) = 0\}$

(13) $\{p(x) : \deg(p(x)) \leq 4,\ \begin{array}{l} p(1) + p''(-1) = 0 \\ p(2) - p'(1) = 0 \end{array}\}$

(14) $\{p(x) : \deg(p(x)) \leq d,\ p(x) = p(-x)\}$

(15) $\{p(x) : \deg(p(x)) \leq d,\ p(x) + p(-x) = 0\}$

(16) $\{p(x) : \deg(p(x)) \leq 3,\ p(x) - 2p'(x) + xp''(-x) = 0\}$