Algebra Lineare – Esercizi del 20/11/08

Date le basi $\mathcal{B}$ e $\mathcal{B}'$ di V e $v \in V$ determinare la matrice di cambio da $\mathcal{B}$ a $\mathcal{B}'$, calcolare $[v]_{\mathcal{B}}$ e $[v]_{\mathcal{B}'}$ e verificare la formula di cambiamento di coordinate:

(1) $V = \mathbb{R}^2 \quad \mathcal{B} = \left(\begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right)$

$\mathcal{B}' = \left(\begin{pmatrix} 4 \\ -1 \end{pmatrix}, \begin{pmatrix} -7 \\ 2 \end{pmatrix} \right) \qquad v = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

(2) $V = \{ x \in \mathbb{R}^3 : 2x_1 - 3x_2 + 5x_3 = 0 \}$

$\mathcal{B} = \left(\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \right) \qquad \mathcal{B}' = \left(\begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \right)$

$v = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$

(3) $V = \mathbb{R}_{\leq 2}[t] \qquad \mathcal{B} = (1+t, 1-t^2, t+2t^2)$

$\mathcal{B}' = (2-t^2, t, 1+t+t^2) \qquad v = 2+t$

(4) $V = \{ x \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0 \}$

$\mathcal{B} = \left(\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \right) \qquad \mathcal{B}' = \left(\begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 1 \\ 1 \end{pmatrix} \right)$

$v = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$

Date le basi $\mathcal{B}$ e $\mathcal{B}'$ di V, $\mathcal{C}$ e $\mathcal{C}'$ di W, e $f: V \to W$ lineare, trovare $[f]^{\mathcal{C}}_{\mathcal{B}}$, $[f]^{\mathcal{C}'}_{\mathcal{B}'}$ e verificare la formula di cambiamento di basi:

(5) $V = \mathbb{R}^3 \quad W = \mathbb{R}^2 \quad f\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_2 + x_3 \\ x_2 - 2x_3 \end{pmatrix}$

$$\mathcal{B} = \left(\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} \right) \quad \mathcal{B}' = \left(\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} \begin{pmatrix} -3 \\ 6 \\ -4 \end{pmatrix} \right)$$

$$\mathcal{C} = \left(\begin{pmatrix} 4 \\ -7 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \end{pmatrix} \right) \quad \mathcal{C}' = \left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 9 \end{pmatrix} \right)$$

(6) $V = \mathbb{R}^2 \quad W = \mathbb{R}_{\leq 1}[t] \quad f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (x_1 + 3x_2) + (2x_1 - x_2)t$

$$\mathcal{B} = \left(\begin{pmatrix} 7 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) \quad \mathcal{B}' = \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right)$$

$$\mathcal{C} = (1 - t, 2 + t) \quad \mathcal{C}' = (2 - t, 1 + t)$$

(7) $V = \{ x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0 \} \quad W = \mathbb{R}^3$

$$\mathcal{B} = \left(\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right) \quad \mathcal{B}' = \left(\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right)$$

$$\mathcal{C} = \left(\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right) \quad \mathcal{C}' = \left(\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$f\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_3 \\ 2x_1 + x_2 \\ 3x_2 - x_3 \end{pmatrix}$$

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(8) Determinare la base $\mathcal{B}$ di $\mathbb{R}^2$ tale che

$$[x]_{\mathcal{B}} = \begin{pmatrix} 3x_1 - 2x_2 \\ 4x_1 + 5x_2 \end{pmatrix} \qquad \forall x \in \mathbb{R}^2$$

(9) Sapendo che $[v]_{\left(\binom{4}{1}\binom{3}{1}\right)} = \binom{-1}{2}$

trovare $[v]_{\left(\binom{-5}{2}\binom{-7}{3}\right)}$.

(10) Determinare la base $\mathcal{B}$ di $\mathbb{R}^2$ tale che

$$[f_A]_{\mathcal{B}}^{\mathcal{E}^{(3)}} = \begin{pmatrix} 3a_{11} + 2a_{12} & 5a_{11} + 3a_{12} \\ 3a_{21} + 2a_{22} & 5a_{21} + 3a_{22} \\ 3a_{31} + 2a_{32} & 5a_{31} + 3a_{32} \end{pmatrix} \quad \forall A \in \mathcal{M}_{3\times 2}$$

(11) Determinare la base $\mathcal{C}$ di $\mathbb{R}^3$ tale che

$$[f_A]_{\mathcal{E}^{(2)}}^{\mathcal{C}} = \begin{pmatrix} 2a_{11} + 3a_{21} + a_{31} & 2a_{12} + 3a_{22} + a_{32} \\ a_{11} - 2a_{21} + 5a_{31} & a_{12} - 2a_{22} + 5a_{32} \\ 3a_{11} + 4a_{21} + 2a_{31} & 3a_{12} + 4a_{22} + 2a_{32} \end{pmatrix}$$

$$\forall A \in \mathcal{M}_{3\times 2}$$

(12) Sapendo che $f : \mathbb{R}^2 \to \mathbb{R}^2$

$$[f]_{\left(\binom{2}{1},\binom{3}{2}\right)}^{\left(\binom{4}{-1}\binom{3}{-1}\right)} = \begin{pmatrix} 5 & 2 \\ 1 & -3 \end{pmatrix}$$

trovare $[f]_{\left(\binom{3}{2},\binom{4}{3}\right)}^{\left(\binom{5}{2},\binom{2}{1}\right)}$.