

Algebra lineare

Esercizi dell'11/12/08

(1) Completare $\begin{pmatrix} 2 \\ i \\ 1+i \\ -1 \end{pmatrix}$ a una base di

$$\{z \in \mathbb{C}^4 : iz_1 + (1+i)z_2 - 2z_3 + (i-3)z_4 = 0\}$$

(2) Estrarre da $\begin{pmatrix} 2-i \\ 3i \\ 2i \\ 1-i \end{pmatrix}, \begin{pmatrix} 5 \\ -3+6i \\ -2+4i \\ 3-i \end{pmatrix}, \begin{pmatrix} 1+2i \\ i \\ 2 \\ 1-i \end{pmatrix}, \begin{pmatrix} 3+i \\ 3+5i \\ 6+2i \\ 2-4i \end{pmatrix}, \begin{pmatrix} -i \\ 1+2i \\ 2-i \\ -2/3 \end{pmatrix}$

una base di $\{z \in \mathbb{C}^4 : 2iz_1 + (i-1)z_2 + (i+1)z_3 + 3z_4 = 0\}$

(3) Calcolare

$$\begin{pmatrix} 2-i & 4i \\ 1+2i & 3+i \end{pmatrix}^{-1} \quad \begin{pmatrix} 1+i & 2+3i \\ 4-i & 1-2i \end{pmatrix}^{-1}$$

$$\begin{pmatrix} 1-i & 2 & -i \\ 3i & 4+i & 0 \\ 1+2i & 1 & 1 \end{pmatrix}^{-1}$$

$$\det \begin{pmatrix} 1+3i & 2 & -i & -1+i \\ 2-i & 1+i & 2 & 1 \\ 1 & 2+i & 2+3i & 0 \\ 2+i & -1 & 2 & i \end{pmatrix}$$

(4) Trovare una presentazione parametrica di

$$\{z \in \mathbb{C}^3 : (1+i)z_1 + 2iz_2 + (3-2i)z_3 = 5-2i\}$$

$$\left\{ z \in \mathbb{C}^4 : \begin{array}{l} iz_1 - 2z_2 + (1+i)z_3 + 3z_4 = 1-i \\ (2+i)z_1 + iz_2 - (2+i)z_3 + 4iz_4 = 3+2i \end{array} \right\}$$

(5) Trovare una presentazione cartesiana ↓

$$\begin{pmatrix} 1+i \\ 2 \\ i-1 \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 2+3i \\ -2i \\ 4+i \end{pmatrix} \right) \subset \mathbb{C}^3$$

$$\begin{pmatrix} 2-3i \\ 4+2i \\ 1-i \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 2 \\ 1-i \\ 3i \end{pmatrix}, \begin{pmatrix} i \\ -3 \\ 2+5i \end{pmatrix} \right) \subset \mathbb{C}^3$$

$$\begin{pmatrix} 1 \\ i \\ 0 \\ 1+i \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 2i \\ -1 \\ 4+3i \\ -5i \end{pmatrix} \right) \subset \mathbb{C}^4$$

$$\begin{pmatrix} 2i \\ -1+i \\ 3 \\ -2+3i \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 1 \\ i \\ 2+i \\ 3-i \end{pmatrix}, \begin{pmatrix} 2+3i \\ -2 \\ 4 \\ 1+i \end{pmatrix} \right) \subset \mathbb{C}^4$$

(6) Risolvere

$$z^3 = 2 - 11i$$

$$z^4 + (7+24i) = 0$$

$$2z^2 + (3-5i)z - 5(1+i) = 0$$

$$6z^2 - (1+5i)z - 2 = 0$$

$$z^3 z + 4i = 2z(z + i\bar{z})$$