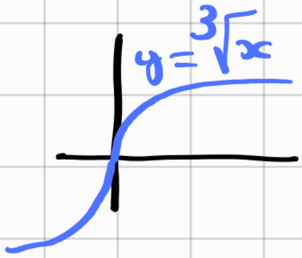
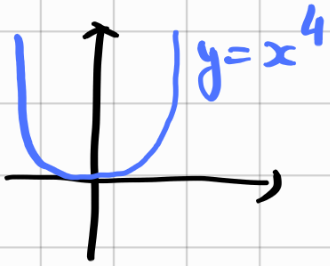
ANALISI MATEMATICA B
LEZIONE 10 – 9.10.2023

ES. Risolvere: $\log_2\left[\sqrt[3]{(x+1)^4-3}-2\right] \le 3$





Sol: $0 < \sqrt[3]{(x+1)^4-3}-2 \le 2^3$

$$2 < \sqrt[3]{(x+1)^4-3} \le 10$$

$$8 < (x+1)^4-3 \le 1000$$

$$11 < (x+1)^4 \le 1003$$

se $x+1 \ge 0$:

$$\sqrt[4]{11} < x+1 \le \sqrt[4]{1003}$$

$$\sqrt[4]{11}-1 < x \le \sqrt[4]{1003}-1$$

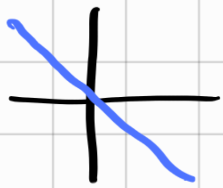
se $x+1<0$:

$$11 < (-x-1)^4 \le 1003$$

$$\sqrt[4]{11} < -x-1 \le \sqrt[4]{1003}$$

$$\sqrt[4]{11}+1 < -x \le \sqrt[4]{1003}+1$$

$$-\sqrt[4]{1003}-1 \le x < -\sqrt[4]{11}-1$$



Finalmente: $-\sqrt[4]{1003}-1 \le x < -\sqrt[4]{11}-1$ o $\sqrt[4]{11}-1 < x \le \sqrt[4]{1003}-1$

$h(x) = f(g(x))$ $\qquad f^{-1}, g^{-1}$

$h^{-1}(y) = g^{-1}(f^{-1}(y))$

Se la x compare una volta sola questo metodo funziona.
Se la x compare due volte:

ES $\qquad 2^x = x^2$.

Insiemi numerici

cardinalità

$\mathbb{N}$: $+ \quad \cdot \quad a^b$ — numerabile.

$\mathbb{Z}$: $-$ — numerabile.

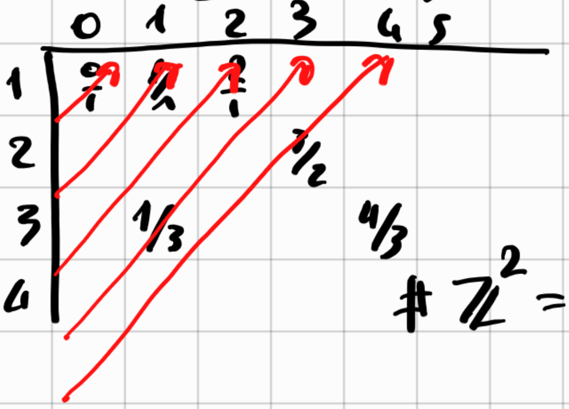
$\mathbb{Q}$: $:$ $\quad \frac{p}{q}$ — numerabile

$0, 1, -1, 2, -2, \frac{1}{2}, -\frac{1}{2}, 3, -3, \frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, \ldots$

salto

$\to \frac{p}{q} \sim (p,q)$

I metodo diagonale di Cantor



$\# \mathbb{Z}^2 = \mathbb{Z}$

$\mathbb{R}$: inf, lim $\qquad \# \mathbb{R} > \# \mathbb{N}$

II metodo diagonale di Cantor $\qquad \#[0,1) > \# \mathbb{N}$

0, 2

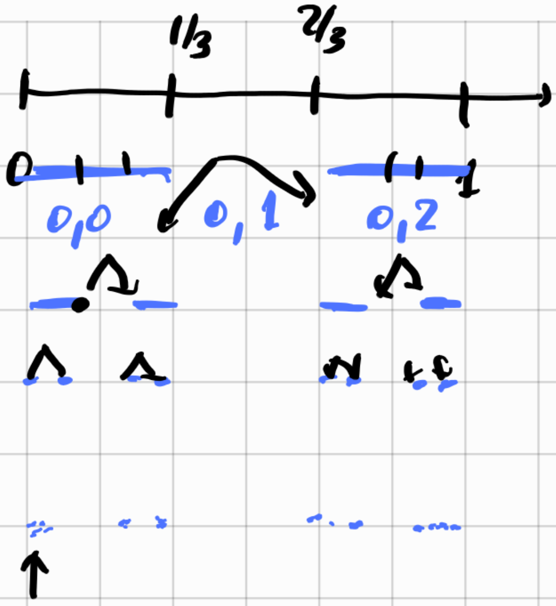
0, 5 7 3 2 7 5 6 7 ...
0, 1 2 3 7 3 7 9 9 2 ...
0, 6 6 0 7 4 9 5 ...
3 2 5

x

$0{,}9999\ldots = 1$.

Insieme di Cantor:

$$C = \frac{C}{3} \cup \frac{C+2}{3}$$



↑

frattale autosimile

polvere di Cantor:

punto fisso

$$\frac{1}{9} - \frac{1}{10} = \frac{1}{90}$$

$\frac{1}{9} = 0.11111$ (in base 10)

$= 0{,}01$ (in base 3)

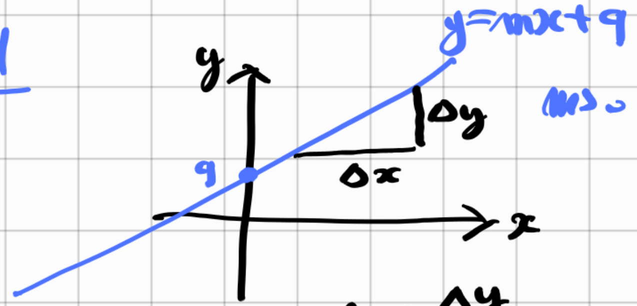
$$\#2^{\mathbb{N}} = \#\mathcal{P}(\mathbb{N}) = \#C \leq \#\mathbb{R}$$

$$\vee \qquad\qquad \wedge$$

$$\#\mathbb{N} = \#\mathcal{P}(\mathbb{Q})$$

FUNZIONI LINEARI

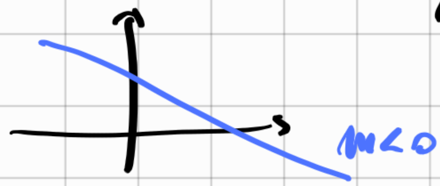
$$f(x) = mx + q$$

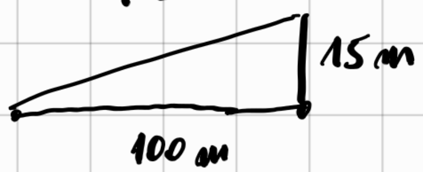


$$m = \frac{\Delta y}{\Delta x}$$

15% = pendenza

$\frac{15}{100}$





% $\quad \frac{1}{100}$

$x+2+3x+7$
$= 4x+9$

FUNZIONI QUADRATICHE

$$f(x) = ax^2 + bx + c.$$

4 la x compare 2 volte.

$$(x+7)^2 = x^2 + 14x + 49$$

↑

$a \neq 0$

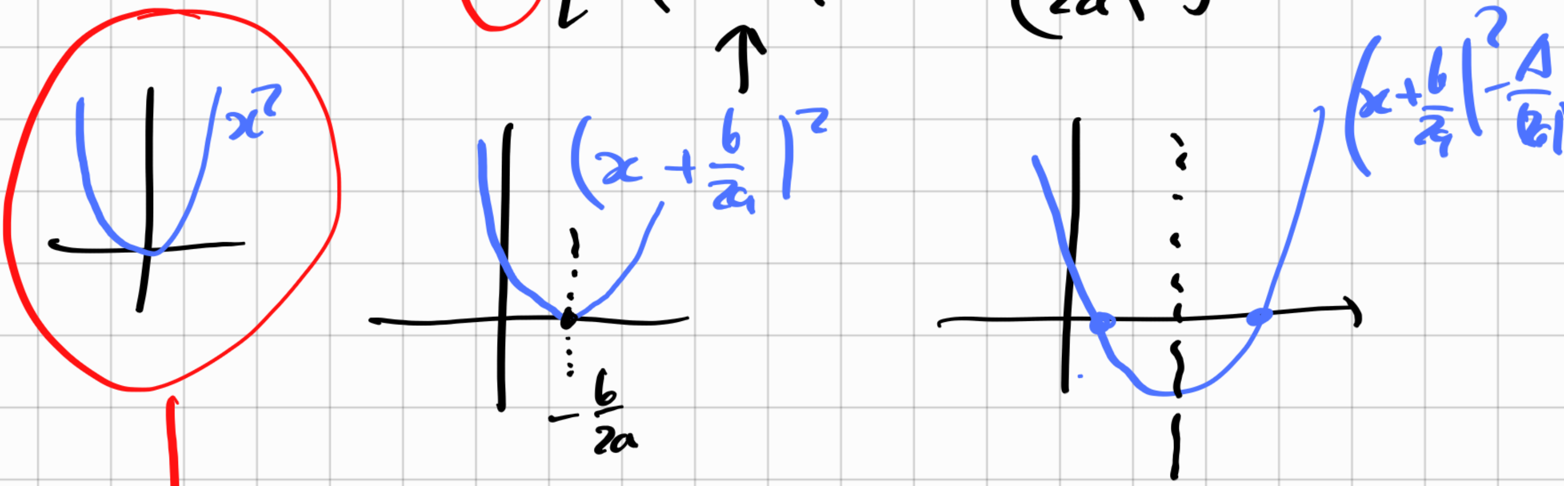
$$ax^2+bx+c = a\left[x^2+\underbrace{\frac{b}{a}x}+\frac{c}{a}\right]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{(2a)^2}\right]$$

$\Delta = b^2-4ac$

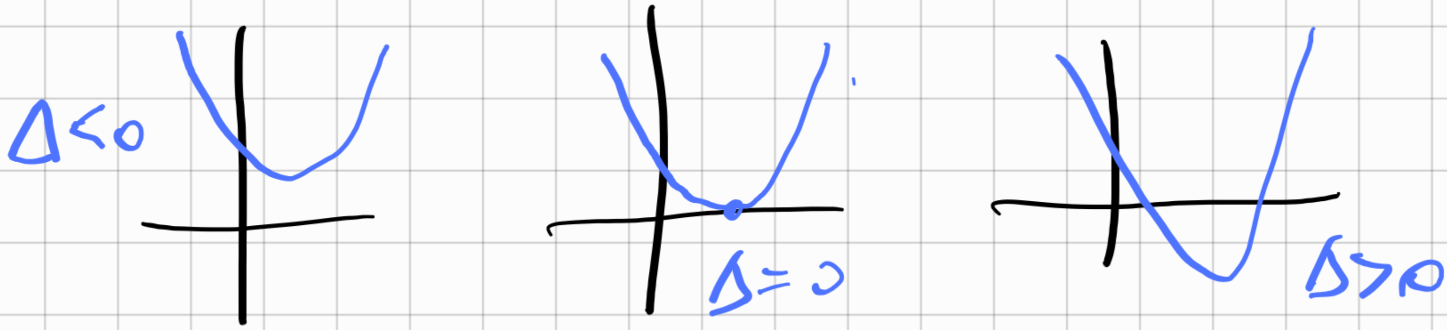
$$= a\left[\left(x+\frac{b}{2a}\right)^2-\frac{\Delta}{(2a)^2}\right]$$



Es: dimostra che $\underline{y=x^2}$ è una parabola

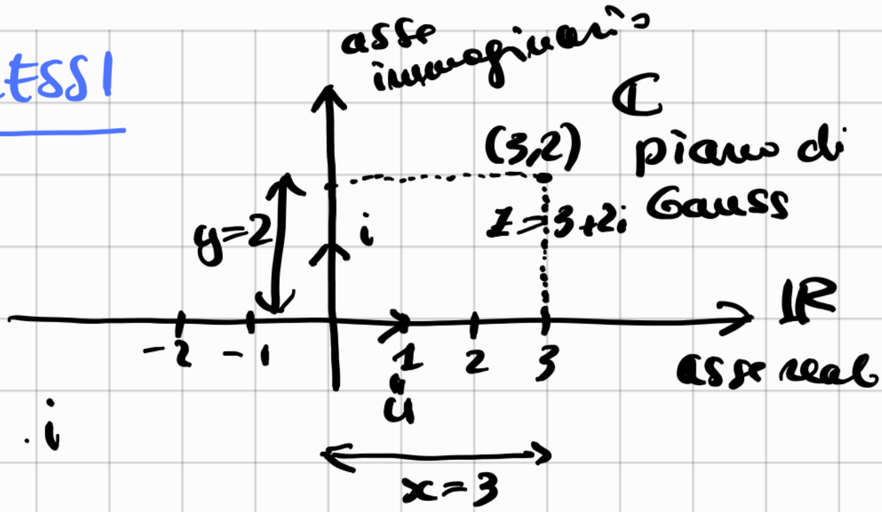
Formula risolutiva eq. II grado:

$$x_{12} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$



NUMERI COMPLESSI

$\mathbb{C}$



$z \in \mathbb{C}$ $\qquad z = x \cdot 1 + y \cdot i$

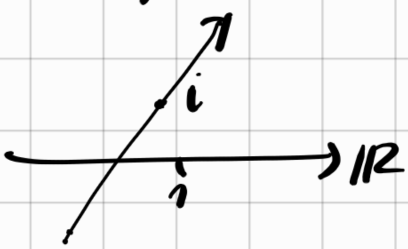
$\qquad\qquad = x + iy$

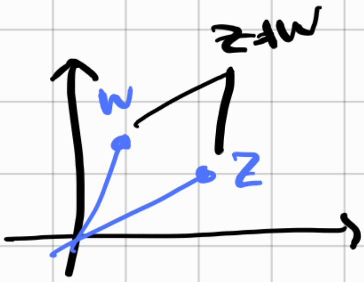
parte reale $\qquad$ parte immaginaria

$x = \mathrm{Re}\, z$ $\qquad y = \mathrm{Im}\, z$

$\qquad\qquad\qquad \in \mathbb{R}.$

$i \notin \mathbb{R}.$





$z = x + iy \qquad w = a + ib$

$z + w = (a+x) + i(b+y).$

$t \in \mathbb{R}. \quad t \cdot z = tx + ity$

$z \cdot w = (x+iy) \cdot (a+ib)$

$= ax + iay + ibx + i^2 by$

$= ax + i^2 by + i(ay + bx)$

$= ax - by + i(ay + bx)$

$\boxed{i^2 = -1}$

$(\mathbb{C}, 0, 1, +, \cdot)$ è un campo.

Tutto banale salvo l'inverso moltiplicativo:

$z \neq 0$

$\frac{1}{z} = ?$

$z = x + iy$

$\bar{z} = x - iy$ ← coniugato

$z\bar{z} = |z|^2 \in \mathbb{R}$

$z \cdot \bar{z} = (x+iy)(x-iy)$

$= x^2 - i^2 y^2 = x^2 + y^2 = |z|^2$

$|z| = \sqrt{x^2+y^2}$

$$z \cdot \underbrace{\frac{\bar{z}}{|z|^2}}_{= \frac{1}{z}} = 1.$$