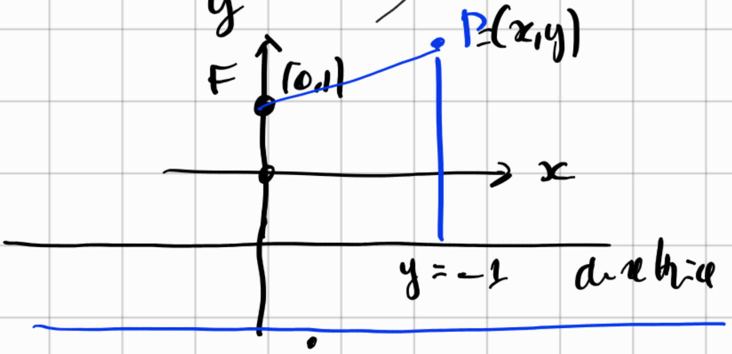
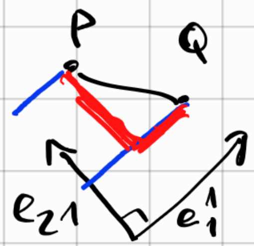
ANALISI MATEMATICA B
LEZIONE 15 - 26.10.2022

Esercizio $\boxed{y = ax^2 + bx + c}$ è una parabola.





$$P = P_x \cdot e_1 + P_y \cdot e_2$$
$$Q = Q_x \cdot e_1 + Q_y \cdot e_2$$

$$P \simeq (P_x, P_y)$$
$$Q \simeq (Q_x, Q_y)$$

$$\boxed{d(P,Q) = \sqrt{(P_x - Q_x)^2 + (P_y - Q_y)^2}}$$

se $|P| = 1$, $|Q| = 1$, $(P,Q) = 0$

$$d(tP, sP) = |t-s|$$

$$\overline{PF} \qquad P$$
$$\| \qquad\qquad \|$$

$$\sqrt{x^2 + (y-1)^2} = |y+1|$$

$$x^2 + (y-1)^2 = (y+1)^2$$

$$x^2 + \cancel{y^2} - 2y + \cancel{1} = \cancel{y^2} + 2y + \cancel{1}$$

$$x^2 = 4y \qquad y = \frac{x^2}{4}$$

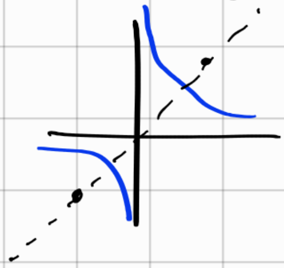
In generale

$$\boxed{y = a\cdot x^2} \longrightarrow \boxed{Y = X^2}$$

scegliendo s

$$\begin{cases} X = s\cdot x \\ Y = s\cdot y \end{cases}$$

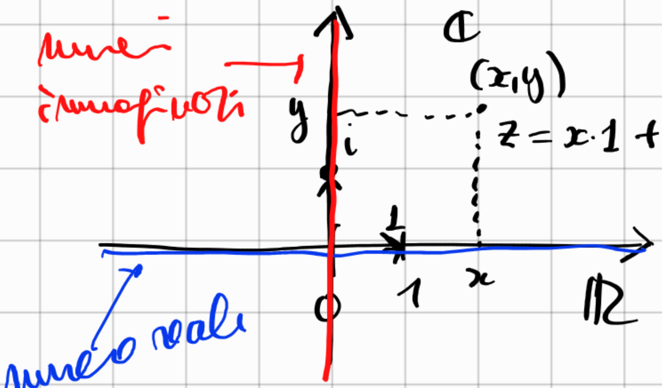
Es Dimostrare che $y = \frac{1}{x}$ è una iperbole con asintoti gli assi cartesiani



NUMERI COMPLESSI

$$ax^2+bx+c=0$$

$$x_{12} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$



$$z = x\cdot 1 + y\cdot i = \boxed{x+iy}$$

$$\sqrt{-10} = \sqrt{10}\sqrt{-1}$$

i

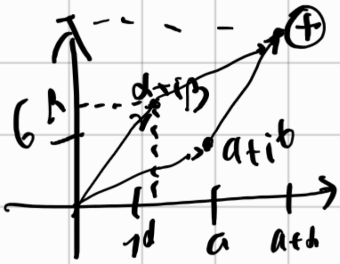
$$\mathbb{C} = \mathbb{R}\times\mathbb{R}$$

$t\in\mathbb{R}$ $\quad t\cdot(x+iy) = tx + i(ty)$

$$(x,y) \rightsquigarrow (tx,ty)$$

$$(a+ib)+(\alpha+i\beta) = (a+\alpha)+i(b+\beta)$$

$$[\,(a,b) + (\alpha,\beta) = (a+\alpha,\ b+\beta)\,]$$



immaginario

$$z = x + iy \qquad x, y \in \mathbb{R}$$

numero complesso — Parte reale — Parte immaginaria

numero reale

$$\operatorname{Re} z = x \qquad \operatorname{Im} z = y$$

$i \cdot y$ è immaginario

Parte reale — Parte immaginaria

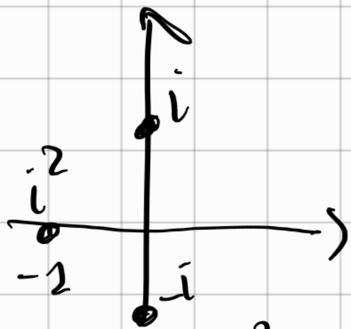
Posso definire una moltiplicazione in modo che

(1) proprietà distributiva $\quad z\cdot(w+v) = z\cdot w + z\cdot v$

(3) $i^2 = -1$ $\qquad$ (2) proprietà commutativa

$$(a+bi)\cdot(\alpha+i\beta) = a\cdot\alpha + ib\alpha + a\cdot\beta\cdot i + b\beta i^2$$

$$= (a\cdot\alpha - b\beta) + i(b\alpha + a\beta)$$



[oss1: se $a = t,\ b = 0$ $\quad t(\alpha + i\beta) = t\alpha + i t\beta$]

[oss2:

$$(-i)^2 = (0 + (-1)\,i)^2$$

$$= (0 + (-1)\cdot i)\cdot(0 + (-1)\cdot i)$$

$$= (0 - 1) + i(0 + 0) = -1$$

$$\boxed{x^2 = 10} \qquad x_1 = \sqrt{10} \qquad x_2 = -\sqrt{10}$$

Si può verificare che $\mathbb{C}$ con $+$ e $\cdot$ è un campo.

$$\overset{0}{+} \quad \overset{1}{\cdot}$$

1\. è elemento neutro: $(1+0i)\cdot(x+iy) = x-0\cdot y+i(0\cdot x+1\cdot y)$
$= x+iy.$

$$t\cdot(x+iy) = tx+ity$$

$\underbrace{\text{vale le proprietà distributive,}}$ commutative.

Se $z \neq 0$ esiste $w \in \mathbb{C}$: $w\cdot z = 1$.

$z = x+iy$

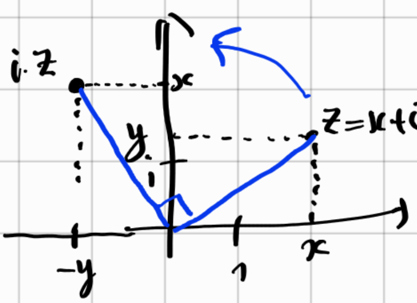
$$(x+iy)\cdot(x-iy) = x^2-(iy)^2 = x^2 - i^2y^2 = x^2+y^2$$

$$\boxed{w = \frac{x-iy}{x^2+y^2}} \qquad z\cdot w = 1.$$

Se $z \neq 0$ $\quad \underbrace{x^2+y^2}>0$

$$w = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

<u>Oss</u> $\quad i\cdot(x+iy) = ix-y = -y+ix$



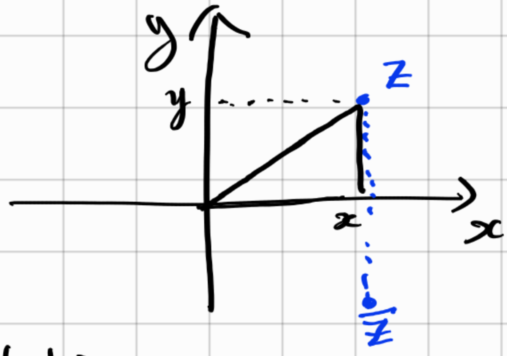
$i\cdot z =$ "ruotato di z di $90°$ in senso antiorario".

<u>Oss</u> $\quad tz =$ "riscalamento di ragione t".

Nuove operazioni su $\mathbb{C}$

Coniugato: $\quad z\in\mathbb{C} \qquad \bar{z}$ o (z^*) $\in\mathbb{C}$

$$\overline{x+iy} = x-iy$$



$z\in\mathbb{R} \iff \bar{z}=z.$

Modulo: $\quad z\in\mathbb{C} \qquad |z|\in\mathbb{R},\ |z|\geq 0$

$$|x+iy| = \sqrt{x^2+y^2}$$ = "distanza di z da 0".

Se $z\in\mathbb{R}$ $(y=0)$ $\quad |x+i0| = \sqrt{x^2+0} = \sqrt{x^2} = |x|.$

modulo $\qquad\qquad$ valore assoluto

Oss $\quad z = x+iy$

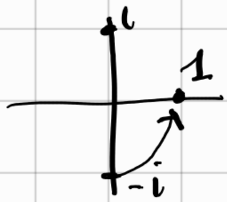
$$z\cdot\bar{z} = (x+iy)\cdot(x-iy) = x^2+y^2 = |z|^2$$

$z\neq 0 \iff |z|\neq 0$, $\iff \bar{z}\neq 0$

$$\frac{z\cdot\bar{z}}{|z|^2} = 1 \qquad \frac{1}{z} = \frac{\bar{z}}{|z|^2}$$

Es. Calcolare $\frac{1}{z}$ per $z\in\{i,\ 1+i,\ 3i+2,\ -7 \ldots\}$

$$\frac{1}{i} = \frac{\bar{i}}{|i|^2} = \frac{-i}{1} = -i$$



Proprietà del coniugio.

$$\overline{z+w} = \bar{z} + \bar{w}$$

$$\overline{z \cdot w} = \bar{z} \cdot \bar{w}$$

$$\bar{\bar{z}} = z$$

$$z \cdot \bar{z} = |z|^2$$

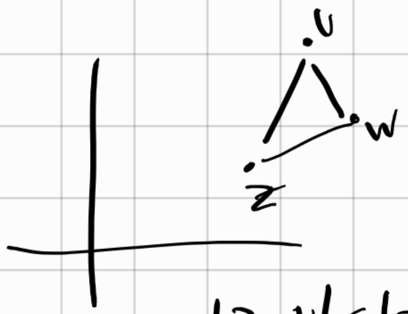
$$\begin{cases} \dfrac{z+\bar{z}}{2} = \operatorname{Re} z \\[2ex] \dfrac{z-\bar{z}}{2i} = \operatorname{Im} z \end{cases}$$

$$|z \cdot w| = |z| \cdot |w|$$

verifica: $|z \cdot w|^2 = z \cdot w \cdot \overline{z w} = z \cdot w \cdot \bar{z} \cdot \bar{w} = z \cdot \bar{z} \cdot w \cdot \bar{w}$

$= |z|^2 \cdot |w|^2 \Rightarrow |z \cdot w| = |z| \cdot |w|.$

$$|z+w| \le |z| + |w|$$



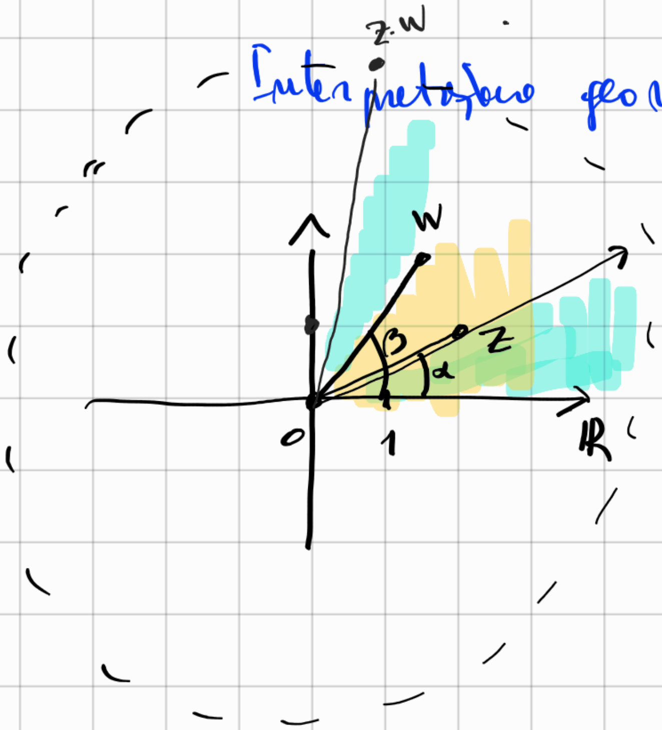
$$|-z| = |z|$$

$$\big||z|\big| = |z|$$

$$|z-u| \le |z-w| + |w-u|$$

distanza tra z e u

Interpretazione geometrica del prodotto.



$$|z \cdot w| = |z| \cdot |w|$$

α = "Arg z" = l'angolo identificato da $\vec{Oz}$ e l'asse reale

$$\operatorname{Arg} z \cdot w = \operatorname{Arg} z + \operatorname{Arg} w$$

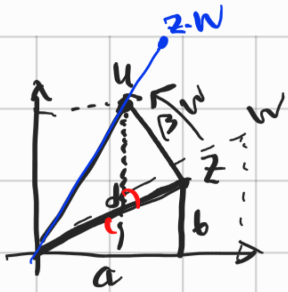


Figura 1.8 sugli appunti

$z \cdot w$

$z = a + ib$

$w = \alpha + i\beta$

$\alpha^2 = a^2 + b^2$

$|R_0 w| = |z|$