ANALISI MATEMATICA B
LEZIONE 4 — 24. 9. 2021

COIMPLICAZIONE LOGICA

$P \Leftrightarrow Q$

P	Q	$P \Leftrightarrow Q$
F	F	V
V	F	F
F	V	F
V	V	V

OPERATORI LOGICI:

$\neg P$	non
$P \wedge Q$	e
$P \vee Q$	o
$P \Rightarrow Q$	se P allora Q
$P \Leftarrow Q$	P se Q
$P \Leftrightarrow Q$	P se e solo se Q

PREDICATI

$P(x)$: $x > 2$ ha un valore di verità che dipende da x

$P(7)$ è una proposizione (vera)

x è una variabile "libera"

$P(x,y)$: $x+2=y$

x, y sono variabili libere.

QUANTIFICATORI LOGICI

- UNIVERSALE

$$\underbrace{\forall x : P(x)}_{x \text{ è una variabile "muta"}}$$

es: $\forall x : x > 2$ è una proposizione FALSA

$x > 2 \wedge \forall z : z < 3$ COSA SIGNIFICA?

- ESISTENZIALE

$\exists x : P(x)$ es: $\exists x : x > 2$ è una proposizione VERA

Posso avere più variabili:

$$\forall x \exists y : P(x,y)$$

es: $R: \forall x \exists y : x+2 > y$

$P(x,y)$: $x+2>y$

$Q(x)$: $\exists y : x+2>y$

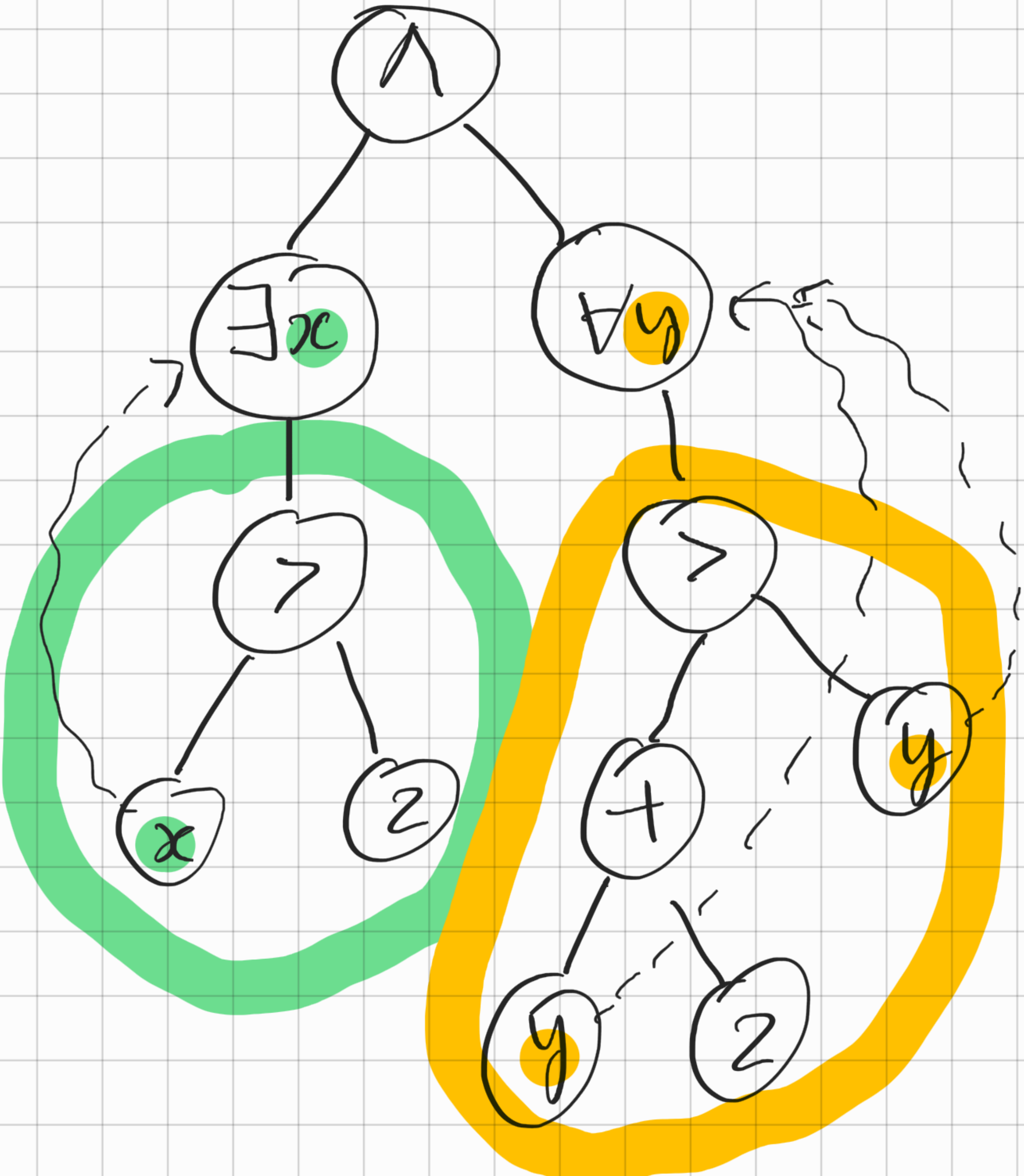
R : $\forall x \exists y : x+2>y$

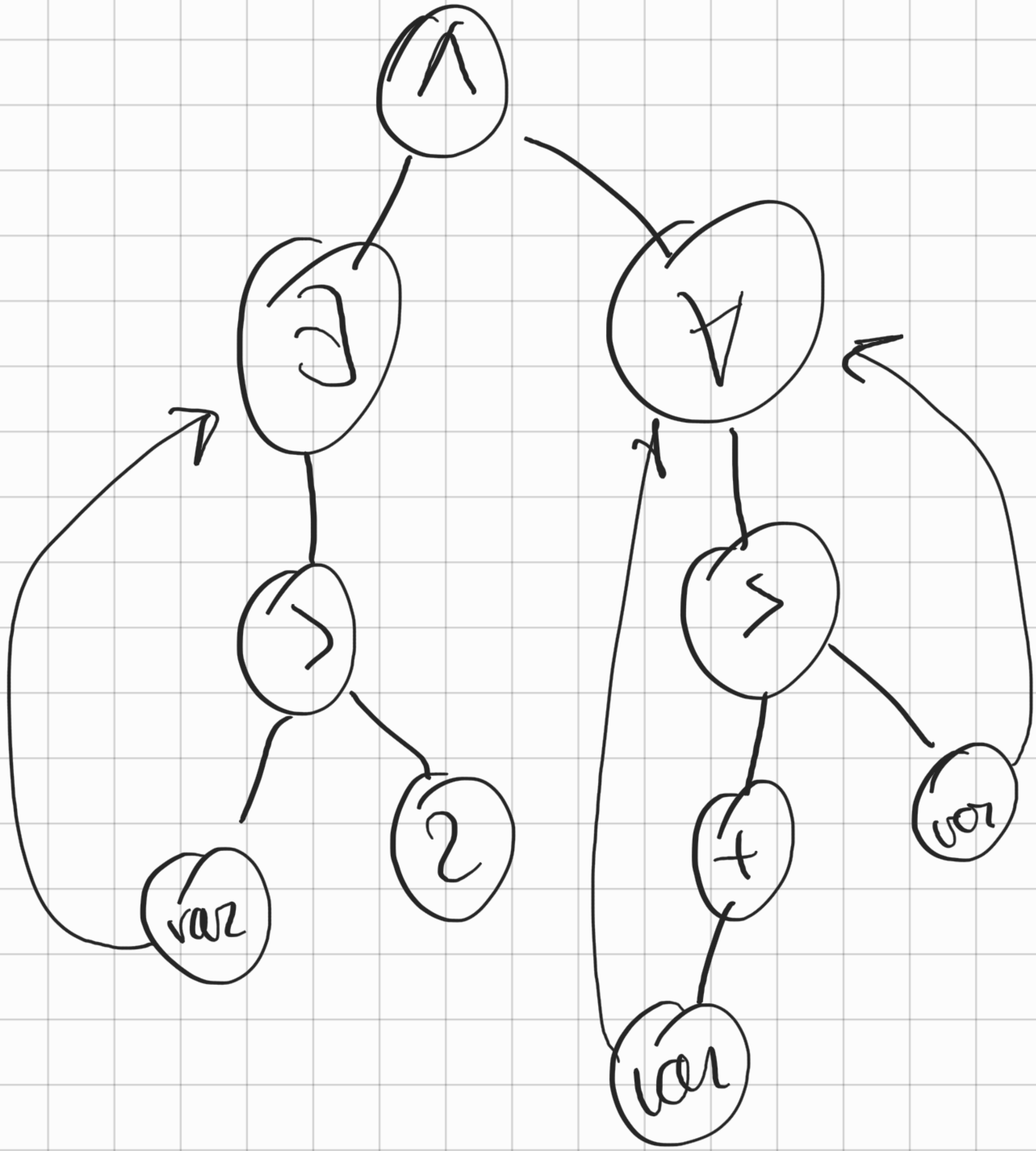
$$R \Longleftrightarrow \forall z \, \exists \alpha : z + 2 > \alpha$$

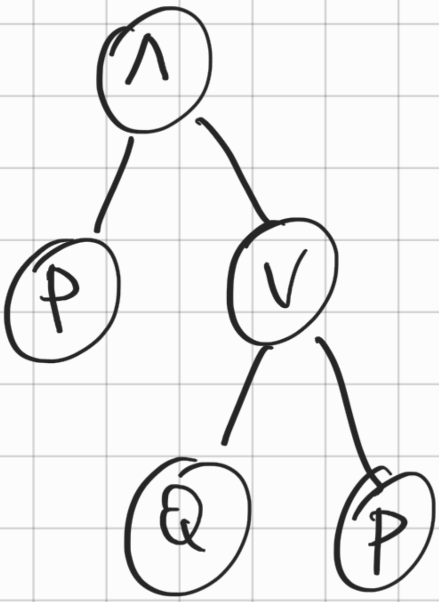
- $$\left(\underbrace{\exists x : x > 2}_{\varphi}\right) \wedge \left(\underbrace{\forall x : x + 2 > x}_{\varphi}\right)$$

$$\Updownarrow$$

$$\left(\exists x : x > 2\right) \wedge \left(\forall y : y + 2 > y\right)$$

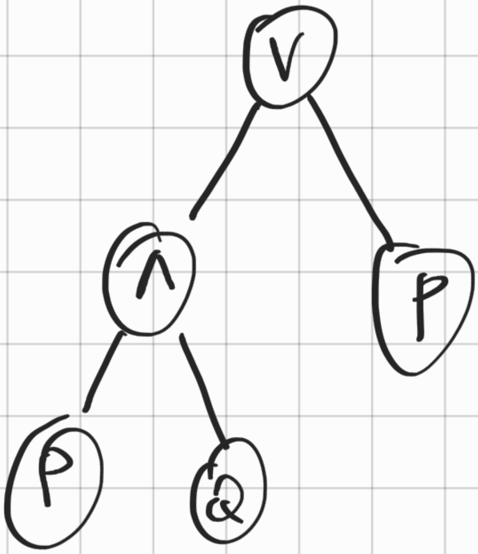
- 





$P \wedge (Q \vee P)$

$$\underbrace{(P \land Q) \lor P}$$



$$\neg \forall x : P(x) \iff \exists x : \neg P(x)$$

$$\neg \exists x : P(x) \iff \forall x : \neg P(x)$$

$$\neg \exists m : B(m)$$

m = mela
B = blu

non esiste una mela blu

$$\forall m : \neg B(m)$$

ogni mela non è blu

$$\neg \exists \qquad \nexists$$

$$\neg (x = y) \qquad x \neq y$$

(1) $\forall x : \exists y : P(x,y)$

$\Uparrow$

(2) $\exists y : \forall x : P(x,y)$

Es

	in $\mathbb{Z}$	in $\mathbb{N}$
$\forall x \exists y : x > y$	V	F
	$\Uparrow$	$\Uparrow$
$\exists y \forall x : x > y$	F	F

Es

	in $\mathbb{Z}$	in $\mathbb{N}$
$\forall x \exists y : x \geq y$	V	V
	$\Uparrow$	$\Uparrow$
$\exists y \forall x : x \geq y$	F	V

$$\exists x \exists y : P(x,y) \Longleftrightarrow \exists y \exists x : P(x,y)$$

$$\forall x \forall y : P(x,y) \Longleftrightarrow \forall y \forall x : P(x,y)$$

Esercizio $\exists y\ \forall x : x \geq y$

$\to \exists y\ \neg \exists x : x < y$

$\to$ falso: basta prendere

$x = y - 1$

$\neg \exists y\ \neg \exists x : x < y$?

$\forall y\ \cancel{\neg \neg} \exists x : x < y$

basta prendere $x = y - 1$

$$\boxed{\forall y : \quad y - 1 < y}$$

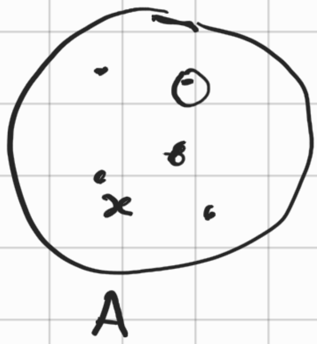
$$\frac{P(7)}{\exists x : P(x)} \qquad \frac{\forall x : P(x)}{P(7)}$$

$\neg \forall x : P(x) \iff \exists x : \neg P(x)$

$\neg \exists x : P(x) \iff \forall x : \neg P(x)$

TEORIA DEGLI INSIEMI

$\boxed{x \in A}$ $\quad$ x è un elemento di A



Socrate $\in$ Uomo

def

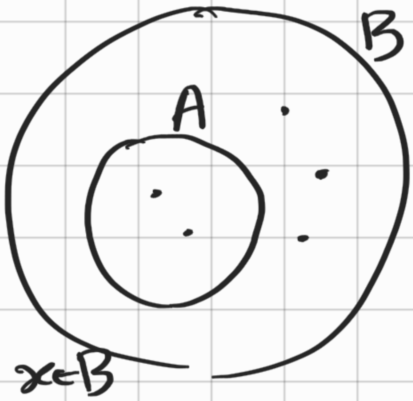
$$A \subseteq B \overset{def}{\iff} \underbrace{\forall x : x \in A \Rightarrow x \in B.}$$

Es $\quad A \subseteq A$ sempre vero.

$$A \supseteq B \overset{def}{\iff} B \subseteq A$$

$$\Updownarrow$$

$$\forall x : x \in A \Leftarrow x \in B$$



def

$$A = B \iff A \subseteq B \wedge B \subseteq A$$

$$\iff \left(\forall x : x \in A \iff x \in B \right)$$

Assioma $\quad$ esiste l'insieme vuoto

$$\exists \emptyset : \neg \exists x : x \in \emptyset$$

Assioma (singoletto) $\quad A = \{x\}$

$$\forall x \, \exists A : \forall y : y \in A \iff x = y$$

A $\boxed{x \in A \land x \notin A}$