LOCAL AND GLOBAL EXISTENCE AND UNIQUENESS OF SMOOTH SOLUTIONS OF A MODIFIED BURGERS EQUATION IN $\mathbb{R}^n$

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Topic #8: *Nonstandard Methods in the study of Navier-Stokes equations and in Mathematical Physics.*

Consider the equation:

$$u_t = \nu\Delta u - (u \cdot \nabla)u - cu + f(x,t) \quad \text{for} \quad x \in [0,1]^n \;\text{ and }\; t \in (0,\infty),$$

together with periodic boundary conditions and initial condition $u(t,0) = g(x)$. As with the Navier-Stokes equations, the major difficulty in existence proofs for this problem is the unbounded advection term, $(u\cdot\nabla)u$.

We study existence and uniqueness of a smooth solution based on a discretization by a suitable Euler scheme for all real values of the parameter c. It is shown that there exists $c_0 > 0$ (dependent only on the Lipshitz constants of g and f) so that the solution exists globally for all $c \geq c_0$. For $c < c_0$ it is shown that the solution exists in an interval $[0,T)$, where $T \leq \frac{1}{K}$, with K depending only on n and the values of the Lipshitz constants of f and g.

We also give a proof of uniqueness of smooth solutions, on domains they are known to exists.

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