WHAT DO NONSTANDARD METHODS TELL US ABOUT FORMAL LAURENT SERIES FIELDS OVER FINITE FIELDS?

FRANZ VIKTOR KUHLMANN

Topic #2: *Nonstandard Methods in Algebra, Algebraic Geometry and Topology*

It is a longstanding open problem in model theoretic algebra whether formal Laurent series fields over finite fields have a decidable elementary theory. In order to attack this problem, knowledge about the structure of the nonstandard models of these fields would be very helpful. For instance, model theorists had thought of a nice recursive axiom system for such fields. But by constructing a nonstandard model of this axiom system which has elementary properties different from those of the Laurent series fields, I was able to show that this axiom system is not complete (J. Symb. Logic 66 (2001), 771-791). Yuri Ershov has introduced the notion of extremality, which is an elementary property satisfied by the Laurent series fields. There is some hope that adding this property to the mentioned axiom system will complete it. But we have very little knowledge about the nonstandard models that satisfy this property. I will sketch Ershov's nonstandard proof for the fact that the Laurent series fields are extremal, and then discuss its limitations and the important open problems about extremality, which has turned out to be an extremely interesting notion in valuation theory.

Mathematical Sciences Group, University of Saskatchewan, Canada
E-mail address: fvk@math.usask.ca, fvk@snoopy.usask.ca