

The Curvature of Envelope of a Family of Straight Lines in a Plane

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Abstract

By using methods of nonstandard analysis given by Robinson, A., and axiomatized by Nelson, E., Through this paper the following problems are studied.

1. We give a nonstandard definition of the envelope of a family of lines $\{L_t\}$ defined in a Projective Homogeneous Coordinate (PHC) by $u(t)X + v(t)Y + w(t)Z = 0$ and generalizing it to the case where the coefficient vector $(u(t), v(t), w(t))$ has a singularity of order $n-1$. Moreover we present applications for conic sections, by searching of the family of lines which has conic sections as an envelope curve.
2. We give the generalized curvature expression of envelope curve of a family of lines as a function of the coefficients u, v and w .

Keywords: Infinitesimals, Envelope, Singularity, Curvature.

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