RELATIVE SET THEORY

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[Topic #1: *Nonstandard Theories and Models, and Foundations of Nonstandard Methods.*]

In a 1904 paper, D. Hilbert wrote, "We know sets before we know their elements." I will argue that coherent account of infinitesimals and other nonstandard objects in the framework of set theory leads to a view that is in accord with this dictum.

The usual presentations of nonstandard analysis postulate a fixed hierarchy of standard, internal, and external sets. Definitions of derivative and other notions from calculus in terms of infinitesimals then make sense only for standard functions and arguments. The desire to extend them to all internal functions and arguments calls for adoption of the relative point of view pioneered by G. Wallet, Y. Peraire and E. Gordon; in this view, any internal set can be treated as standard.

First I will present an elementary framework for calculus with relative infinitesimals and some examples of proofs in it; this is an ongoing joint project with R. O'Donovan. This framework constitutes a fragment of FRIST, an extension of Peraire's internal set theory RIST. After giving examples in FRIST where multiple levels of standardness are essential, I will describe a further extension of FRIST to a theory (GRIST) that is "complete over ZFC" in a technical sense.

D. Ballard advocated the "general relativistic" conception according to which it should be possible to treat even external sets as standard. I will conclude by outlining a theory in the language of a ternary membership predicate "x is an element of y relative to w" where this conception is fully implemented. In Relative Set Theory, every set w determines its associated standard universe S_w and internal universe I_w, both of which satisfy ZFC − Regularity. I_w with the induced levels of standardness satisfies GRIST. The theory has no need of classes; any collection W definable relative to w is again a set (in general not in S_w) and has its own associated universe S_W, where power set, replacement and choice are freely available.

The main technical result is: RST is a conservative extension of ZFC.

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