ON A POSSIBLE SET OF AXIOMS FOR THE EXTERNAL NUMBERS OF NONSTANDARD ANALYSIS

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In practice we do not always need to be exact. We may ignore small quantities and be happy with approximations. Thus, important questions arise on the propagation of errors: when are certain orders of magnitude small enough to be ignored? In order to answer these questions the external numbers of Nonstandard Analysis have been proposed.

This paper is meant to be a first attempt to extend the commutative field axioms of $\mathbb{R}$ to the external numbers. We present a list of axioms which enable us to define an algebraic structure, called assembly, which generalizes the algebraic notion of group. We show that the set of axioms in consideration is satisfied by $(\mathbb{R}, +)$ and the external set of external numbers.

The main obstacle in combining an additive and multiplicative assembly within a single structure is formed by the distributive law with respect to subtraction. We present a first alternative and discuss some examples.

References

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