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5.2. For the numerical solution of the problem

$$y' = \lambda(y - \sin t) + \cos t, \quad y(0) = 1, \quad 0 \le t \le 1,$$

whose exact solution is $y(t) = e^{\lambda t} + \sin t$, consider using the following four two-step methods, with $y_0 = 1$ and $y_1 = y(h)$ (i.e., using the exact solution so as not to worry here about y_1).

(a) ~~Your unstable method from the previous question.~~

$$y_n = 4y_{n-1} - 3y_{n-2} - 2hf_{n-2}$$

(b) The midpoint two-step method

$$y_n = y_{n-2} + 2hf_{n-1}.$$

(c) Adams–Bashforth

$$y_n = y_{n-1} + \frac{h}{2}(3f_{n-1} - f_{n-2}).$$

(d) BDF

$$y_n = \frac{(4y_{n-1} - y_{n-2})}{3} + \frac{2h}{3} f_n.$$

Consider using $h = .01$ for $\lambda = 10$, $\lambda = -10$, and $\lambda = -500$. Discuss the expected quality of the obtained solutions in these twelve calculations. Try to do this without calculating any of these solutions. Then confirm your predictions by doing the calculations.

5.3. Write a program which, given k and the values of *some* of the coefficients $\alpha_1, \alpha_2, \ldots, \alpha_k$, $\beta_0, \beta_1, \ldots, \beta_k$ of a linear k-step method, will

- find the rest of the coefficients, i.e., determine the method, such that the order of the method is maximized,
- find the error coefficient C_{p+1} of the leading local truncation error term.

Test your program to verify the second and the last rows in each of the Tables 5.1 and 5.2.

Now use your program to find C_{p+1} for each of the six BDF methods in Table 5.3.

5.4. Write a program which, given a linear multistep method, will test whether the method is

- 0-stable,
- strongly stable.

[Hint: This is a very easy task using MATLAB, for example.]

Use your program to show that the first six BDF methods are strongly stable, but the seven-step and eight-step BDF methods are unstable. (For this you may want to combine your program with the one from the previous exercise.)

5.5. The famous Lorenz equations provide a simple example of a chaotic system (see, e.g., [92, 93]). They are given by

$$\mathbf{y}' = \mathbf{f}(\mathbf{y}) = \begin{pmatrix} \sigma(y_2 - y_1) \\ ry_1 - y_2 - y_1y_3 \\ y_1y_2 - by_3 \end{pmatrix},$$