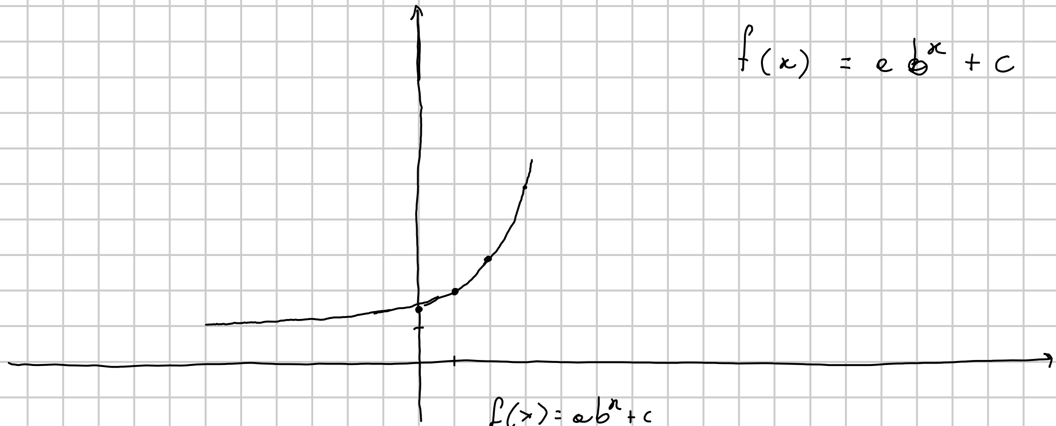
LEZIONE 8 NOVEMBRE

ES. 38.

$f: \mathbb{R} \to \mathbb{R}$



$f(x) = a b^x + c$

Determinare a, b, c.

$f(x) = ab^x + c$ 　　　 Dal grafico.

$x = 1 \quad ab + c = f(1) = 2$

$x = 2 \quad a b^2 + c = f(2) = 3$

$x = 3 \quad a b^3 + c = f(3) = 5$

$$\begin{cases} ab + c = 2 \\ ab^2 + c = 3 \\ ab^3 + c = 5 \end{cases} \qquad \begin{cases} c = 2 - ab \\ ab^2 + 2 - ab = 3 \\ ab^3 + 2 - ab = 5 \end{cases}$$

$$\begin{cases} ab^2 - ab = 1 \\ ab^3 - ab = 3 \end{cases} \qquad \begin{cases} ab(b-1) = 1 \\ ab(b^2-1) = 3 \end{cases}$$

$$\begin{cases} ab = \frac{1}{b-1} \\ \frac{1}{b-1}(b^2-1) = 3 \end{cases} \qquad \begin{cases} ab = \frac{1}{b-1} \\ \frac{\cancel{(b-1)}(b+1)}{\cancel{b-1}} = 3 \end{cases}$$

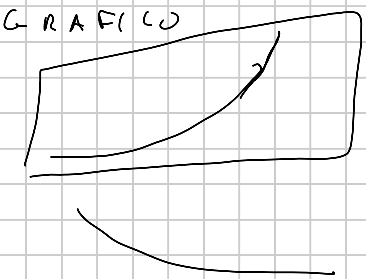
$$\begin{cases} b + 1 = 3 \\ ab(b-1) = 1 \end{cases} \qquad \begin{cases} b = 2 \\ 2a \cdot 1 = 1 \qquad a = \frac{1}{2} \\ c = 2 - ab = 2 - 2 \cdot \frac{1}{2} = 1 \end{cases}$$

$a = \frac{1}{2} \qquad b = 2 \qquad c = 1 \qquad f(x) = \frac{1}{2} 2^x + 1$

$f(x) = a\,b^x + c$

$f(x) = \underline{a}\,\underline{b}^x + \underline{c}$ voglio sapere $\lim_{x \to -\infty} f(x)$

$\lim_{x \to -\infty} f(x) = 1$ DAL GRAFICO



$$\lim_{x \to -\infty} b^x \begin{cases} 0 & b>1 \\ +\infty & b<1 \end{cases}$$

$$\lim_{x \to -\infty} \underset{\downarrow \atop a}{a}\;\underset{\downarrow \atop 0}{b^x} + \underset{\downarrow \atop c}{c} = a \cdot 0 + c = c = 1$$

$f(1) = ab + c = \underline{ab + 1 = 2}$ $\quad ab = 1$

$f(2) = ab^2 + c = ab^2 + 1 = 3$ $\quad ab^2 = 2$

$\underline{b = 2}$

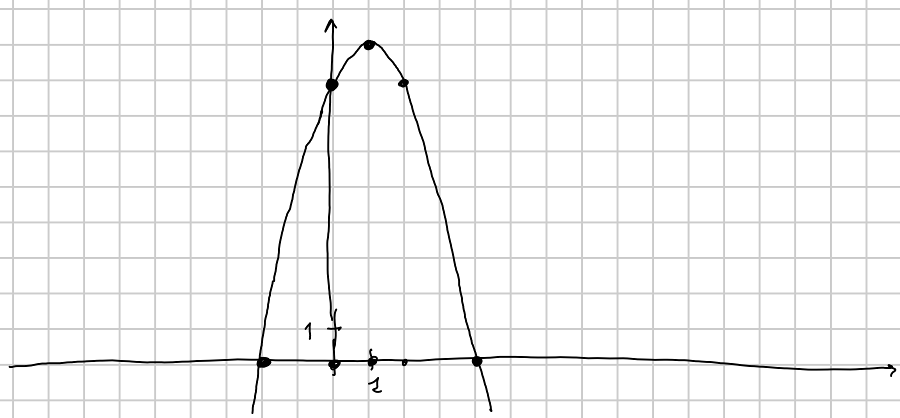
<u>ES. 37</u>

$\underline{f(x) = ax^2 + bx + c}$

$f(0) = 8$

$f(1) = 9$

$f(2) = 8$



$$\begin{cases} f(0) = 8 = a\cdot 0 + b\cdot 0 + c = c \\ f(1) = 9 = a\cdot 1 + b\cdot 1 + c = a+b+c \\ f(2) = 8 = a\cdot 2^2 + b\,2 + c = 4a+2b+c \end{cases} \qquad c = 8$$

$$\begin{cases} \cancel{9}^{1} = a + b \cancel{+8}^{0} \\ \cancel{8}^{0} = 4a + 2b \cancel{+8}^{0} \end{cases} \qquad \begin{cases} a + b = 1 \\ 4a + 2b = 0 \end{cases}$$

$$\begin{cases} b = -2a \\ a - 2a = 1 \end{cases} \qquad \begin{cases} b = -2a \\ a = -1 \end{cases} \qquad \begin{cases} a = -1 \\ b = 2 \\ c = 8 \end{cases}$$

$$f(x) = -x^2 + 2x + 8 \qquad \boxed{\cancel{ax^2} + bx + c} \qquad a\left(x^2 + \underbrace{\frac{b}{a}}_{=B}x + \underbrace{\frac{c}{a}}_{=C}\right)$$

$$f(x) = a\,\boxed{(x^2 + Bx + C)} \rightarrow \text{SI ANNULLA IN } \boxed{x = -2 \text{ E IN } x = 4}$$

$$\underline{(x-4)}\,\underline{(x+2)} = x^2 - 2x - 8$$

$$f(x) = a\,(x^2 - 2x + 8) \qquad f(0) = 8 \qquad a = -1$$

ES. 41

FUNZIONE COSTANTE CHE VALE λ. $\qquad f' \quad Df$

$$g,\ f: \mathbb{R} \to \mathbb{R} \quad e \quad x \quad \lambda \in \mathbb{R}$$

1) $$D(\lambda \cdot f) = D(\lambda)\, f + \lambda\, D(f) = 0 \cdot f + \lambda\, D(f) = \lambda D(f)$$

2) $$D\left(\frac{1}{f}\right) = \frac{D(1)\, f - 1\, D(f)}{f^2} = \frac{0 \cdot f - D(f)}{f^2} = -\frac{D(f)}{f^2}$$

3) $$D(f - g) = D(f + (-1)\cdot g) = D(f) + D(\underline{(-1)\cdot g}) = D(f) - D(g)$$

ES. 39

$$f(x) = 5x^3 + x^2$$

$$D(5x^3 + x^2) = D(5x^3) + D(x^2) =$$

$$= 5\,D(x^3) + D(x^2) = 5 \cdot 3 \cdot x^2 + 2x$$
$$= 15x^2 + 2x$$

$$D(x^\alpha) = \alpha\, x^{\alpha - 1}$$

- $$D\left(\frac{x+1}{x+2}\right) = \frac{D(x+1)\,(x+2) - (x+1)\,D(x+2)}{(x+2)^2} =$$

$$= \frac{\big(D(x) + D(1)\big)(x+2) - (x+1)\big(D(x) + D(2)\big)}{(x+2)^2} =$$

$$= \frac{(1+0)\,(x+2) - (x+1)\,(1+0)}{(x+2)^2}$$

$$= \frac{\cancel{x} + 2 - \cancel{x} - 1}{(x+2)^2} = \frac{1}{(x+2)^2}$$

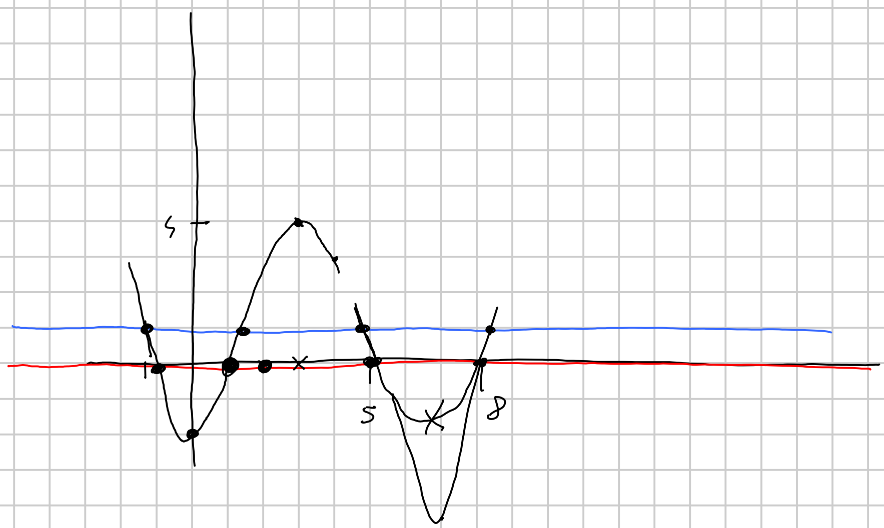
1) $f(1) = 0$

$f(3) = 4$

$f(5) = 0$

$f(7)$ circa $-4{,}7$

2) Determina le soluzioni dell'equazione $f(x) = 1$ e $f(x) = 0$.



$f(x) = 0$ per $x = -1,\ 1,\ 5,\ 8$

$f(x) = 1$ $\quad x = -1{,}2 \quad 1{,}4 \quad 4{,}6 \quad 8{,}1$

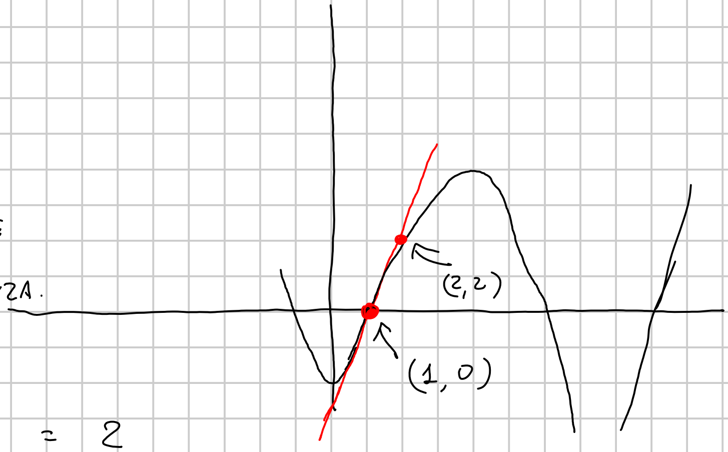
3) Determinare $f'(1)$ $\quad f'(2)$ $\quad f'(3)$ $\quad f'(4)$

$f'(1)$

TRACCIO A OCCHIO

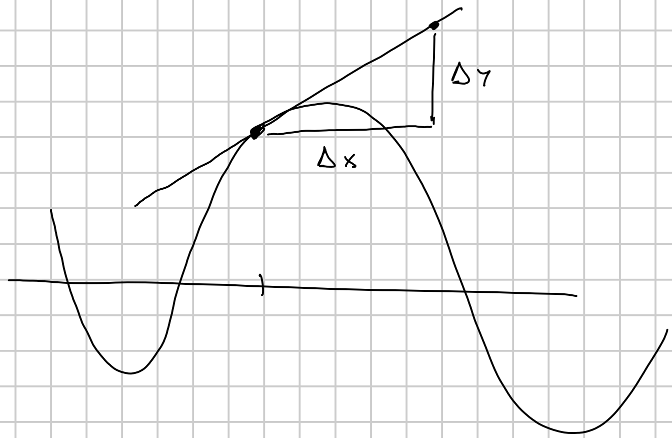
LA RETTA TANGENTE

E NE CALCOLO LA PENDENZA.



$$\frac{\Delta y}{\Delta x} = \frac{2-0}{2-1} = 2$$

4) CAL



4) DETERMINARE L'EQUAZIONE DELLA RETTA TANGENTE PASSANTE PER IL PUNTO DEL GRAFICO CON $x = 1$ $\quad P = (1, f(1)) = \boxed{(1, 0)}$

LA PENDENZA È 2 $\qquad y = 2x + b$

INOLTRE $\qquad 0 = 2 \cdot 1 + b \qquad \boxed{b = -2}$

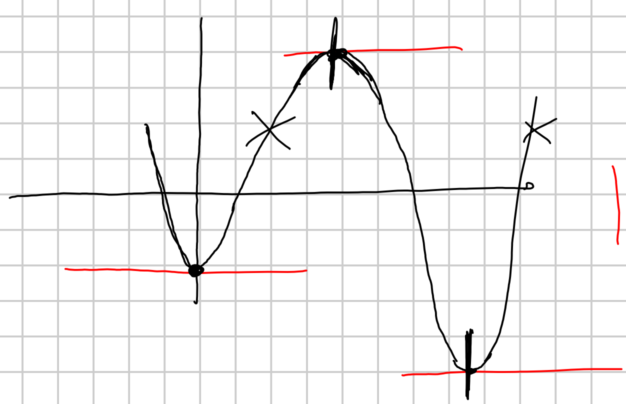
$$\boxed{y = 2x - 2}$$

5) PER QUALI VALORI DI x $\quad f'(x) = 0$.

$x = -0,1$

$x = 3,2$

$x = 6,8$



6) ~~PER~~ IN QUALI INTERVALLI LA DERIVATA È NEGATIVA?

$(-\infty, -0,1) \qquad (3,2,\ 6,8)$

PER QUALI VALORI DI $f'(x) = -0,5$

SE $f'(x) = -0,5$ VUOL DIRE CHE LA PENDENZA DELLA RETTA TANGENTE NEL PUNTO $(x, f(x))$ È $-0,5$

OVVERO È DELLA FORMA $y = -0,5x + b$

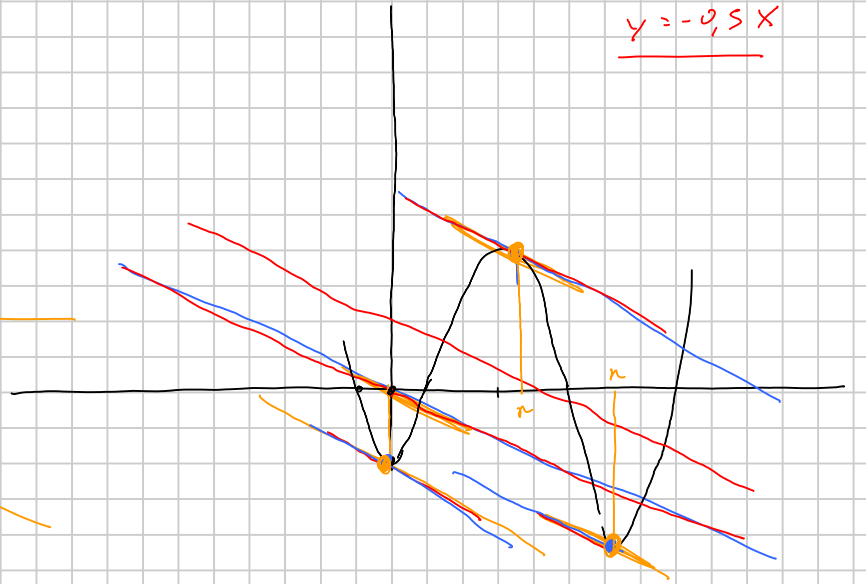
$b = 0$

$y = -0,5X$

$x = -0,3$

$x = 3,4$

$x = 6,5$



DERIVATA DELLA FUNZIONE COMPOSTA

$$h(x) = e^{3x^2+x}$$

$$\underline{x \longrightarrow 3x^2+x} = \underline{y \longrightarrow e^y}$$

$h(2) \qquad 3 \cdot 2^2 + 2 = 14 \qquad e^{14} = h(2)$

$f(x) = 3x^2 + x \qquad g(y) = e^y$

$$h(x) = g(\underbrace{f(x)})$$

SI DICE CHE h È LA COMPOSIZIONE DELLA FUNZIONE f e DELLA FUNZIONE g

$f(x) = 3x^2 + x$ $\qquad g(y) = e^y$

$f'(x) = 3 \cdot 2x + 1$ $\qquad g'(y) = e^y$

$= 6x + 1$

VOGLIO CALCOLARE LA DERIVATA DELLA FUNZIONE h

<u>TEOREMA</u> Se $h(x) = g(f(x))$

$$h'(x) = f'(x)\, g'(y) \qquad \text{DOVE } y = f(x)$$

<u>NELL'ESEMPIO:</u> $h(x) = e^{3x^2+x}$

$f(x) = 3x^2 + x$ $\qquad f'(x) = 6x+1$

$g(y) = e^y$ $\qquad g'(y) = e^y$

$$h'(x) = f'(x) \cdot g'(y)$$

$$= (6x+1) \cdot e^y = (6x+1)\, e^{f(x)}$$

$$= (6x+1)\, e^{3x^2+x}$$

<u>ESR</u>

$$h(x) = \log(\underbrace{x^2+x})$$

$f(x) = \underline{x^2+x}$ $\qquad g(y) = \log y$

$$h(x) = g(f(x))$$

$h'(x) = f'(x)\, g'(y)$ $\qquad$ con $y = f(x)$

$= (2x+1)\, \dfrac{1}{y}$ $\qquad = x^2 + x$

$$= (2x+1)\frac{1}{y} = \frac{2x+1}{x^2+x}$$

ESR

- $\sin(e^x)$ $\qquad D(\sin) = \cos$
- $e^{\sin(x)}$
- 2^x
- $e^{\sin(e^x)}$

$$h(x) = \sin(e^x)$$

$$f(x) = e^x \qquad g(y) = \sin(y)$$

$$h(x) = g(f(x))$$

$$h'(x) = f'(x)\; g'(y) \qquad y = f(x)$$

$$= e^x \cdot \cos(y) = e^x \cos(e^x)$$

$$h(x) = e^{\sin(x)}$$

$$f(x) = \sin(x) \qquad g(y) = e^y$$

$$h'(x) = f'(x)\; g'(y)$$

$$= \cos(x)\; e^y = \cos(x)\; e^{\sin(x)}$$

$$h(x) = 2^x \qquad 2 = e^{\log_e 2}$$

$$2^x = \left(e^{\log_e 2}\right)^x = e^{(\log_e 2)\cdot x} \qquad \left(e^b\right)^c = e^{bc}$$

$f(x) = \underline{(\log_e 2)} \cdot x$ $\qquad$ $g(y) = e^y$

$h'(x) = (\log_e 2) \cdot e^y = (\log_e 2) \cdot \underbrace{e^{(\log_e 2)\cdot x}}$

$= (\log_e 2) \cdot 2^x$

e^x

• $h(x) = e^{(\sin(e^x))} = e^{f(x)}$ con $f(x) = \sin(e^x)$

$x \longrightarrow e^x$

$y \longrightarrow \sin y = z \longrightarrow e^z.$

$h(x) = g(f(x))$ con $f(x) = \underline{\sin(e^x)}$

$g(y) = e^y$ $\qquad$ $g'(y) = e^y$

$h'(x) = f'(x) \; \underline{\underline{g'(y)}} = f'(x) \; e^{\sin(e^x)}$

$= \underline{e^x \cos(e^x)} \; e^{\sin(e^x)}$

• $\log_{10} x$ $\qquad$ • $e^{(\log x)^2}$