Plenary Lectures

Loewner Matrices

Rajendra Bhatia, Indian Statistical Institute
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Mon 9:00, Auditorium

Let f be a smooth function on $\mathbb{R}$. The divided difference matrices whose (i,j) entries are

$$\left[\frac{f(\lambda_i)-f(\lambda_j)}{\lambda_i-\lambda_j}\right]$$

$\lambda_1,\ldots,\lambda_n \in \mathbb{R}$ are called *Loewner matrices*. In a seminal paper published in 1934 Loewner used properties of these matrices to characterise operator monotone functions. In the same paper he established connections between this matrix problem, complex analytic functions, and harmonic analysis. These elegant connections sent Loewner matrices into the background. Some recent work has brought them back into focus. In particular, characterisation of operator convex functions in terms of Loewner matrices has been obtained. In this talk we describe some of this work. The talk will also serve as an introduction to some more recent and more advanced topics being presented by some other speakers in this conference.

Matrices and Indeterminates

Richard A. Brualdi , University of Wisconsin - Madison, USA
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Thu 9:00, Auditorium

An expository talk will be given on matrices some of whose entries are indeterminates over a field. The talk will include some recent joint work on such matrices with Zejun Huang and Xingzhi Zhan, and some recent joint work on combinatorial batch codes with K.P. Kiernan, S.A. Meyer, and M.W. Schroeder, some of which can be placed in the context of such matrices.

Potential Stability and Related Spectral Properties of Sign Patterns

Pauline van den Driessche, University of Victoria, B.C. Canada
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Mon 14:00, Auditorium

An $n \times n$ sign pattern $\mathcal{S}=[s_{ij}]$ has $s_{ij}\in\{+,-,0\}$ and gives rise to an associated sign pattern class of matrices

$$Q(\mathcal{S})=\{A=[a_{ij}]: a_{ij}\in\mathbb{R},\ \text{sign}\ a_{ij}=s_{ij}\ \forall i,j\}.$$

Sign pattern $\mathcal{S}$ has inertia (n_+,n_-,n_0) with $n_++n_-+n_0=n$ if there exists a matrix $A\in Q(\mathcal{S})$ with this inertia. In particular $\mathcal{S}$ is potentially stable if it allows inertia $(0,n,0)$, i.e., there exists a matrix $A\in Q(\mathcal{S})$ with each eigenvalue having a negative real part. Since its introduction in the context of qualitative economics over 40 years ago, the problem of characterizing potential stability of sign patterns remains unsolved except for special classes of sign patterns, e.g., when the digraph associated with the pattern can be represented by a tree. Elaborating on [1], known necessary or sufficient conditions for potential stability are reviewed, techniques for constructing potentially stable patterns described and open problems stated. Some results are also given for sign patterns that allow more general inertias, and those that allow any spectrum of a real matrix.

[1] M. Catral, D.D. Olesky and P. van den Driessche, Allow problems concerning spectral properties of sign patterns, Linear Algebra and its Applications, 430, pp. 3080-3094, 2009.

Computing the Action of the Matrix Exponential, with an Application to Exponential Integrators

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Tue 14:00, Auditorium

A new algorithm is developed for computing $e^{tA}B$, where A is an $n\times n$ matrix and B is $n\times n_0$ with $n_0\ll n$. The algorithm works for any A, its computational cost is dominated by the formation of products of A with $n\times n_0$ matrices, and the only input parameter is a backward error tolerance. The algorithm can return a single matrix $e^{tA}B$ or a sequence $e^{t_kA}B$ on an equally spaced grid of points t_k. It uses the scaling part of the scaling and squaring method together with a truncated Taylor series approximation to the exponential. It determines the amount of scaling and the Taylor degree using the recent analysis of Al-Mohy and Higham [*SIAM J. Matrix Anal. Appl.* 31 (2009), pp. 970-989], which provides sharp truncation error bounds expressed in terms of the quantities $\|A^k\|^{1/k}$ for a few values of k, where the norms are estimated using a matrix norm estimator. Shifting and balancing are used as preprocessing steps to reduce the cost of the algorithm. Numerical experiments show that the algorithm performs in a numerically stable fashion across a wide range of problems, and analysis of rounding errors and of the conditioning of the problem provides theoretical support. Experimental comparisons with two Krylov-based MATLAB codes show the new algorithm to be sometimes much superior in terms of computational cost and accuracy. An important application of the algorithm is to exponential integrators for ordinary differential equations. It is shown that the sums of the form $\sum_{k=0}^{p}\varphi_k(A)u_k$ that arise in exponential integrators, where the φ_k are related to the exponential function, can be expressed in terms of a single exponential of a matrix of dimension $n+p$ built by augmenting A with additional rows and columns, and the algorithm of this paper can therefore be employed.

Joint work with Awad H. Al-Mohy (University of Manchester)

Nonsymmetric Algebraic Riccati Equations Associated with M-matrices: Theoretical Results and Algorithms

B. Meini, University of Pisa, Italy
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Fri 14:00, Auditorium

Nonsymmetric algebraic Riccati equations (NARE) are nonlinear matrix equations of the kind

$$C+XA+DX-XBX=0,$$

where the unknown X is an $m\times n$ matrix and A,B,C,D are matrices of appropriate size. We focus the attention on NAREs whose block coefficients are such that the matrix

$$M=\begin{bmatrix} A & -B \\ C & D \end{bmatrix}$$

is either a nonsingular M-matrix, or a singular irreducible M-matrix. This class of equations arises in a large number of applications, ranging from fluid queues models to transport theory. The solution of interest is the minimal nonnegative one, i.e., the nonnegative solution $X_{\min}$ such that $X_{\min} \leq X$ for any other nonnegative solution X, where the ordering is component-wise.

In this talk we present theoretical properties of the NARE and numerical methods for the computation of the minimal nonnegative solution $X_{\min}$. Particular emphasis is given to the properties of the invariant subspaces, and to the techniques used to trasform the eigenvalues of a pencil, keeping unchanged the invariant subspaces. Concerning numerical methods, special attention is addressed to structure-preserving iterative algorithms; connections between the cyclic reduction algorithm and the structure-preserving doubling algorithm (SDA) are pointed out.

Potpourri of Quasiseparable Matrices

Vadim Olshevsky (LAMA Speaker), University of Connecticut, USA
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Fri 9:00, Auditorium

In this talk we provide a survey of recent results on quasiseparable matrices in three different areas. We start with CMV matrices that garner a lot of attention in the orthogonal polynomials community. Our quasiseparable approach allows one to generalize some already clasical results to a wider class of matrices. The second topic is application of quasiseparable matrices to new digital filter structures. Again, quasiseparable approach allows one to generalize the celebrated Markel-Grey filter structure and Kimura structure. Finally, we desribe the results of error analysis of several published quasiseparable system solvers that indicate that only one of them is a provably backward stable algorithm while the others are not.

Joint work with Tom Bella, Forilan Dopico, Gil Strang and Pavel Zhlobich

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Moments, model reduction and nonlinearity in solving linear algebraic problems

Z. Strakoš, Charles University, Prague, Czech Republic
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Thu 14:00, Auditorium

Krylov subspace methods play an important role in many areas of scientific computing, including numerical solution of linear algebraic systems arising from discretisation of partial differential or integral equations. By their nature they represent *model reductions based on matching moments.* Such view naturally complements, in our opinion, the standard description using the projection processes framework, and it shows their highly nonlinear character.

We present three examples that link algebraic views of problems with views from related areas of mathematics:

- Matching moments reduced order modeling in approximation of large-scale linear dynamical systems is linked with the classical work on moments and continued fractions by Chebyshev and Stieltjes, and with development of the conjugate gradient method by Hestenes and Stiefel.
- We show that Gauss-Christoffel quadrature for a small number of quadrature nodes can be highly sensitive to small changes in the distribution function, and we relate the sensitivity of Gauss-Christoffel quadrature to the convergence properties of the CG and Lanczos methods in exact and in finite precision arithmetic.
- Based on the method of moments, we show how the information from the Golub-Kahan iterative bidiagonalization can be used for estimating the noise level in discrete ill-posed problems.

Joint work with I. Hnětynková (Charles University, Prague), D. P. O'Leary (University of Maryland), M. Plešinger (Technical University, Liberec), P. Tichý (Academy of Sciences, Prague)

Modifications to block Jacobi with overlap to accelerate convergence of iterative methods for banded matrices.

Daniel B. Szyld , Temple University, Philadelphia, USA
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Tue 9:00, Auditorium

Classical Schwarz methods and preconditioners subdivide the domain of a partial differential equation into subdomains and use Dirichlet or Neumann transmission conditions at the artificial interfaces. Optimizable Schwarz methods use Robin (or higher order) transmission conditions instead, and the Robin parameter can be optimized so that the resulting iterative method has an optimal convergence rate. The usual technique used to find the optimal parameter is Fourier analysis; but this is only applicable to certain domains, for example, a rectangle.

In this talk, we present a completely algebraic view of Optimizable Schwarz methods, including an algebraic approach to find the optimal operator or a sparse approximation thereof. This approach allows us to apply this method to any banded or block banded linear system of equations, and in particular to discretizations of partial differential equations in two and three dimensions on irregular domains. This algebraic Optimizable Schwarz method is in fact a version of block Jacobi with overlap, where certain entries in the matrix are modified.

With the computable optimal modifications, we prove that the Optimizable Schwarz method converges in two iterations for the case of two subdomains. Similarly, we prove that when we use an Optimizable Schwarz preconditioner with this optimal modification, the underlying Krylov subspace method (e.g., GMRES) converges in two iterations. Very fast convergence is attained even when the optimal operator is approximated by a sparse transmission matrix. Numerical examples illustrating these results are presented.

Joint work with Martin Gander and Sébastien Loisel (University of Geneva)

Linear algebraic foundations of the operational calculi

Luis Verde-Star , Universidad Autónoma Metropolitana, Mexico City, Mexico
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Wed 9:00, Auditorium

One of the most common problems in Applied Mathematics consists in finding solutions f of a linear functional equation of the form $w(L)f = g$, where w is a polynomial, L is an operator, and g is a known function. Differential and difference equations with constant or variable coefficients are included among such equations.

We will construct a vector space $\mathcal{F}$ of formal Laurent series generated by a set $\{p_k : k \in \mathbb{Z}\}$, with the natural multiplication induced by $p_k p_n = p_{k+n}$, and an operator L defined by $Lp_0 = 0$ and $Lp_k = p_{k-1}$ for $k \neq 0$. Then we will show how to solve the general equation $w(L)f = g$ in the space $\mathcal{F}$ using only elementary linear algebraic ideas.

It turns out that $\mathcal{F}$ is a sort of universal model for the solution of many types of linear functional equations. Giving a suitable particular meaning to the generators p_k we obtain a concrete space where L becomes a given differential-like operator, and the multiplication in $\mathcal{F}$ becomes a "convolution" in the concrete space. For example, if $p_k = t^k/k!$ then L becomes differentiation with respect to t. We can also obtain operators of the form $u(t)D + v(t)I$, and difference operators.

Our development clarifies how the basic concepts of the operational calculi appear in a natural way. For example, we explain the connection between convolutions and divided differences, the role played by quasi-polynomials, and how the transform methods become unnecessary in most cases. Our results generalize some of the ideas presented in [1].

[1] L. Verde-Star, An algebraic approach to convolutions and transform methods, Adv. in Appl. Math. **19**, 117–143, 1997.

Special Lectures

Krylov Subspace Approximations of the Action of Matrix Functions for Large-Scale Problems

Oliver Ernst, TU Bergakademie Freiberg, Germany
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Tue 9:45, Auditorium

We present an overview of recent progress in the development of Krylov subspace methods for the approximation of expressions of the form $f(A)b$, where A is a large, sparse or structured matrix, f is a function such that $f(A)$ is defined and b a given vector. Such an action of a matrix function on a vector is a fundamental computational task in many applications, of which the most prominent is the matrix exponential occurring in initial value problems for systems of ordinary differential equations or semidiscretized partial differential equations. Special emphasis will be given to restarting techniques [1], rational Krylov subspace approximations [2], error estimates and convergence theory. The performance of these techniques will be illustrated for a large-scale problem arising in geophysical exploration [3].

[1] M. Eiermann and O. G. Ernst, A restarted Krylov subspace method for the evaluation of matrix functions. SIAM J. Numer. Anal. 44:2481–2504 (2006)
[2] S. Güttel. Rational Krylov Methods for Operator Functions, doctoral thesis, TU Bergakademie Freiberg (2010).
[3] R. Börner, O. G. Ernst and K. Spitzer. Fast 3D simulation of transient elec- tromagnetic fields by model reduction in the frequency domain using Krylov subspace projection. Geophys. J. Int., 173:766–780 (2008).

Joint work with M. Afanasjew, S. Güttel, M. Eiermann (TU Bergakademie Freiberg)

Zeros of entire functions: from René Descartes to Mark Krein and beyond

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Fri 9:45, Auditorium

The central question of many classical investigations, going back to Descartes, Newton, Euler, and others, is finding zeros of entire and meromorphic functions, given some standard representation of such a function, e.g., its coefficients in some standard basis. Special questions of this type include zero localization with respect to a given curve (e.g., stability and hyperbolicity), behavior of zeros under special maps (e.g., differentiation, Hadamard product), and relations among roots of function families (e.g., orthogonal polynomials). The point of this talk is to give an overview of matrix and operator methods in this area, emphasizing beautiful old and new connections between algebra and analysis. The novel results in this talk are joint with Mikhail Tyaglov.

Multilinear Algebra and its Applications

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Mon 9:45, Auditorium

In mathematics, the study of multilinear algebra is largely limited to properties of a whole space of tensors — tensor products of k vector spaces, modules, vector bundles, Hilbert spaces, operator algebras, etc. There is also a tendency to take an abstract coordinate-free approach. In most applications, instead of a whole space of tensors, we are often given just a single tensor from that space; and it usually takes the form of a hypermatrix, i.e. a k-dimensional array of numerical values that represents the tensor with respect to some coordinates/bases determined by the units and nature of measurements. How could one analyze this one single tensor then?

If the order of the tensor $k = 2$, then the hypermatrix is just a matrix and we have access to a rich collection of tools: rank, determinant, norms, singular values, eigenvalues, condition number, pseudospectrum, RIP constants, etc. This talk is about the case when $k > 2$.

We will see that one may often define higher-order analogues of common matrix notions rather naturally: tensor ranks, hyperdeterminants, tensor norms (Hilbert-Schmidt, spectral, Schatten, Ky Fan, etc), tensor eigenvalues and singular values, etc. We will discuss the utility as well as difficulties of various tensorial analogues of matrix problems. In particular we shall look at how tensors arise in a variety of applications including: computational complexity, control engineering, holographic algorithms, mathematical biology, neuroimaging, numerical analysis, quantum computing, signal processing, spectroscopy, and statistics. Time permitting, we will also describe a few exciting recent breakthoughs, most notably Landsberg's settlement of the border rank of 2×2 matrix multiplications and Friedland's resolution of the Salmon conjecture.

Evolution of MATLAB

Cleve Moler, The MathWorks
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Thu 9:45, Auditorium

We show how MATLAB has evolved over the last 25 years from a simple matrix calculator to a powerful technical computing environment. We demonstrate several examples of MATLAB applications. We conclude with a few comments about future developments, including Parallel MATLAB.

Cleve Moler is the original author of MATLAB and one of the founders of the MathWorks. He is currently chairman and chief scientist of the company, as well as a member of the National Academy of Engineering and former president of the Society for Industrial and Applied Mathematics.
See `http://www.mathworks.com/company/aboutus/founders/clevemoler.html`.

Linear ALgebra Meets Lie Algebra

BERESFORD N. PARLETT, University of California, Berkeley, USA
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Wed 9:45, Auditorium

We examine the matrix congruence class $A \to GA\text{inv}(G)$ in which are preserved not only the eigenvalues of A but the eigenvalues of all the leading principal submatrices of A. We show connections both to the Kostant-Wallach theory in Lie Algebra and to the recent work of Olshevsky, Zhlobich, and Strang on Green's matrices. This is joint work with Noam Shomron.