An introduction to the mathematical modeling of the biological neuron
Part 2: the McCulloch-Pitts model and an interpretation of the "Mozart Effect" by using the trion model

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Lesson 2

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What we will learn today?

- The McCulloch-Pitts model
- Some notes on the “Mozart Effect” (this section will be developed on Monday)
- The trion model

Models of biological neurons

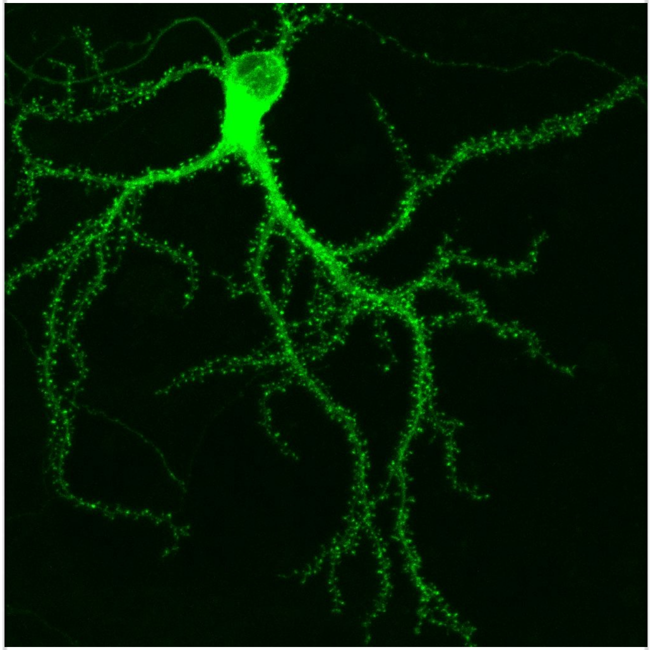
The goal is to model single biological model, or groups of neurons on the basis of their structure and/or behavior

Examples:

- the Hodgkin - Huxley (HH) model

- the **McCulloch - Pitts (MP) model**

- the **“trion” model**



Hodgkin-Huxley model

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_l (V - V_l), \tag{26}$$

where

$$dn/dt = \alpha_n (1 - n) - \beta_n n, \tag{7}$$

$$dm/dt = \alpha_m (1 - m) - \beta_m m, \tag{15}$$

$$dh/dt = \alpha_h (1 - h) - \beta_h h, \tag{16}$$

MP model

Historically, MP is the first model of the neuron: *"A logical calculus of the ideas immanent in nervous activity"* was published in the Bulletin of Mathematical Biophysics in 1943, a decade before the HH model

McCullock was a neuropsychologist and he tried to construct a logic of transitive verbs (1919), and of propositions (1923)

"My object, as a psychologist, was to invent a least psychic event, or 'psychon', that would have the following properties: first, it was to be so simple an event that it either happened or else it did not happen. Second, it was to happen only if its bound cause had happened-shades of Duns Scotus!—that is, it was to imply its temporal antecedent. Third it was to propose this to subsequent psychons. Fourth, these were to be compounded to produce the equivalents of more complicated propositions concerning their antecedents ... In 1921 it dawned on me that these events might be regarded as the all-or-nothing impulses of neurons, combined by convergence upon the next neuron to yield complexes of propositional events."

In 1942, McCullock started his collaboration with Pitts, then a 17-year-old student of logic at the Chicago University

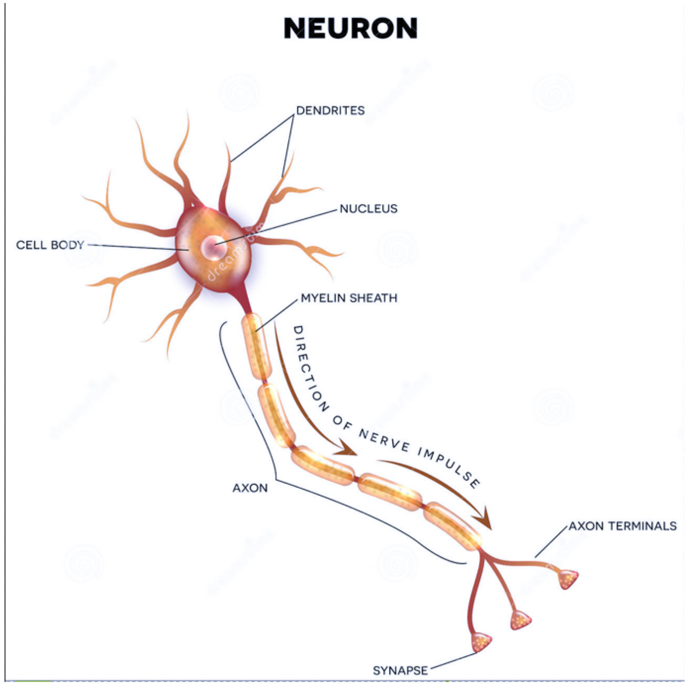
The idea: to consider neurons as computational elements

MP model: key aspects

The neuron's **cell body** can be interpreted as an **elementary computing**

The **neurons** as **input** and the next neuron as **output**

MP neurons are **binary**: they take as input and produce as output **only 0 or 1**



MP model: key aspects

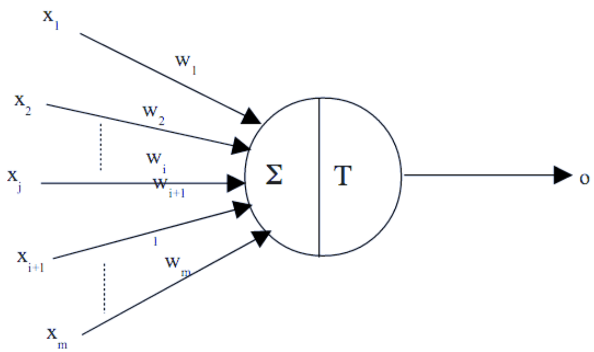
Each neuron has a “voltage” threshold

The threshold associated to a neuron is represented as a non negative number, T

Rule: signals from neurons are summed and outputs 1 if the threshold T is reached and 0 if not

Each input neuron is considered as a variable $x_i \in \{0,1\}$; $x_i = 1$ represents that the neuron is active (able to stimulate), $x_i = 0$ the absence of activation

The number of neurotransmitters which can be released represents the "efficacy" of the synapse; this **efficacy** can be represented as a value w_i, called **weight**, associated to each input neuron x_i

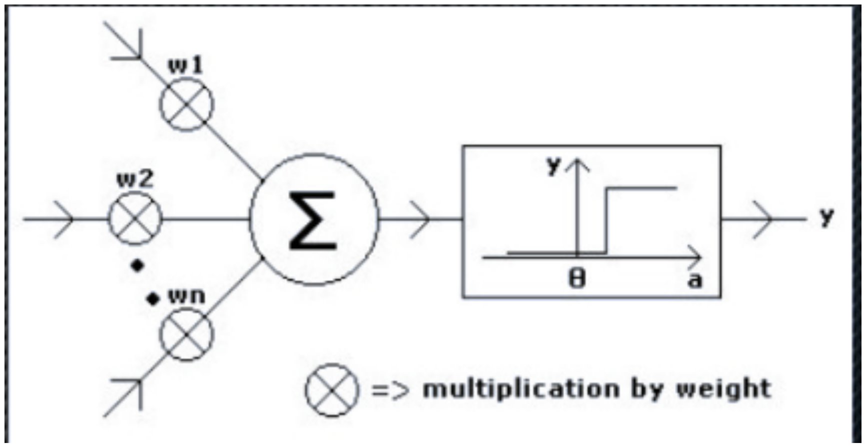


The neuron output is associated to a variable $o \in \{0,1\}$

Mathematical formulation

$o = \theta(a)$

where $a = \sum_i w_i x_i - T$, $i = 1, \ldots, n$ where i is the number of input neurons



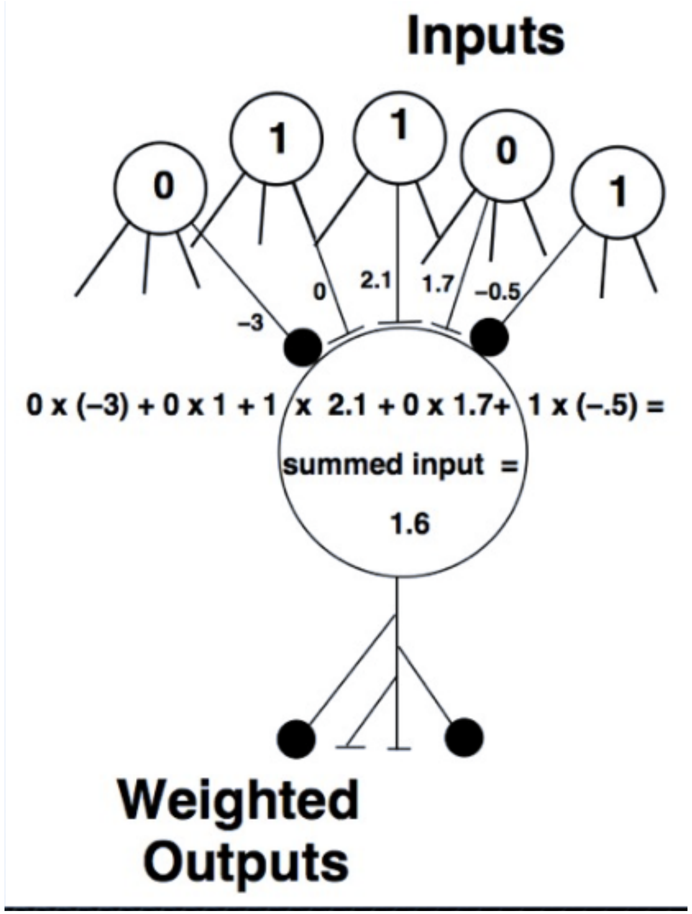
$\theta(x)$ is a step function such that:

$\theta(x) = 1$ if $x > 0$

otherwise $\theta(x) = 0$

Example

- State: the degree of activation of a single neuron (0 or 1)
- Weight: the strength of the connections between 2 neurons ("efficacy" of neurotransmitters)
- Rule: determines how inputs are translated into the output
- What about the output if T = 3? And for T = 1,5?

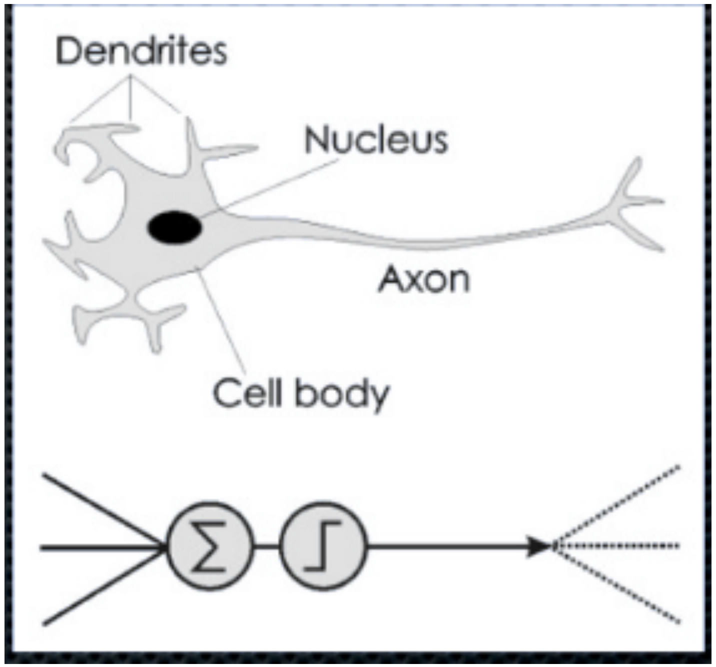


MP vs. HH

HH accurately models the single action potential, while MP how neurons could be combined to do logical operations

MP model is an extremely simplified representation of the neuron but any finite logical proposition can be expressed by a network of MP neurons

When the first neural networks were modeled, the prevailing belief was that intelligence was based on reasoning, and that logic was the foundation of reasoning



How can we obtain the "NOT"?

By properly choosing w_1 and T

For example: $w_1 = -1$ and

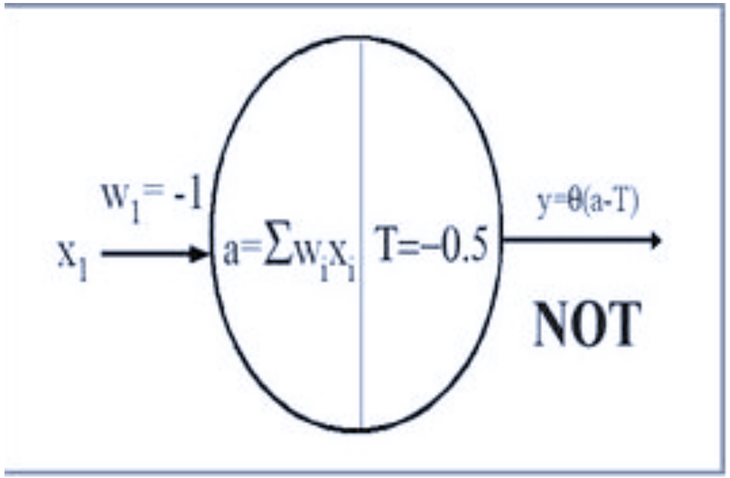
$T = -0,5$

If $x_1 = 0$, $a = 0 > -0,5$, so

$o = 1$ (the "NOT" of 0)

If $x_1 = 1$, $a = -1 < -0,5$, so

$o = 0$ (the "NOT" of 1)



In the **HH** model, the neuron acts as a **circuit with resistances and conductor**

In the **MP** model, inputs come from the output of the other neurons, as **transistors in a circuit**

Like the transistor:

- MP neuron has a threshold T that needed to be reached in order to activate the unit

- the output is stereotyped, being a binary 1 if threshold is reached and remaining at resting value at binary 0 otherwise

Furthermore, in a networks of MP neurons, the inputs come from the outputs of other MP neurons, just as the inputs to a transistor come from other transistors

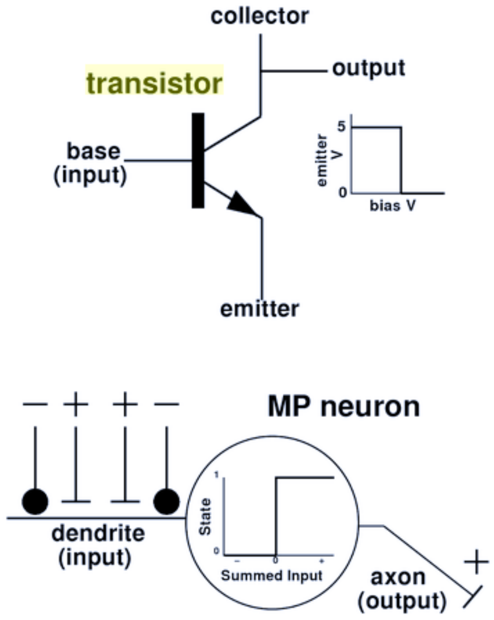


Fig. 6.1: Comparison of transistor and McCulloch-Pitts neuron. Standard symbols are used: a perpendicular line segment is excitatory (+) and a filled circle is inhibitory (−).

Probably, **MP** model has been more influential with **computer scientists** (Von Neumann became intrigued by this model), while **HH** model with **physiologists**

MP neuron is a static model, while HH model analyzes neuronal dynamics

MP is good for exploring computational properties while HH captures realistic important details of neuronal activity

HH is realistic but much complex; although ODE solvers can be applied directly to the system, it would be intractable to compute temporal interactions between many neurons

The "Mozart Effect"

The first model of neuron picked up the most significant features of a biological neuron: all-or-none output resulting from a transfer function applied to a sum of inputs

Subsequent neuronal models evolved where inputs and outputs were real-valued, and the threshold was replaced by a linear or non-linear functions

We will not study the network of MP neurons, but we will use the **Shaw's trion**, a basic unit of the neural network model of the neocortex, to explain the **Rauscher-Shaw's "Mozart Effect"**

A profound dilemma of historical origin is the similarity among such higher brain functions as music, mathematics and chess

For example, original, creative results have been obtained before the age of puberty by Gauss in mathematics, Mozart and Rossini in music

The "Mozart Effect" refers to an enhancement of performance or a change in neurophysiological activity associated with listening to Mozart music, in particular the K448 sonata

Mozart's Sonata for Two Pianos in D Major (K448)

- It is composed by Wolfgang Amadeus Mozart in 1781, when he was 25
- It is written in strict sonata-allegro form
- It was composed for a performance he would give with fellow pianist Josephine von Aurnhammer
- This is one of his few compositions written for two pianos
- The piece typically takes about 25 minutes to be performed.

Mozart's Sonata for Two Pianos in D Major (K448)

The sonata is written in 3 movements:

Allegro con spirito

Andante in G major

Molto Allegro

The 1st movement begins in D major, and sets the tonal center with a strong introduction

The 2 pianos divide the main melody for the exposition, and when the theme is presented both play it simultaneously

The Mozart Effect

In 1993, Rauscher and Shaw published a letter in "Nature" that demonstrated a temporary improvement in performance on abstract spatial reasoning tests after 10 min of listening to Mozart's sonata K448

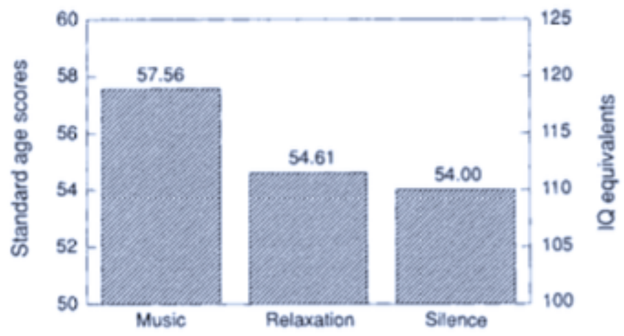
The improvement was shown to last for no longer than 15 minutes after the listening period, and was only tested for spatial-temporal reasoning

This startling finding became known as the "Mozart Effect", and has since been explored by several research groups

An exciting public debate has arisen over the music of Mozart

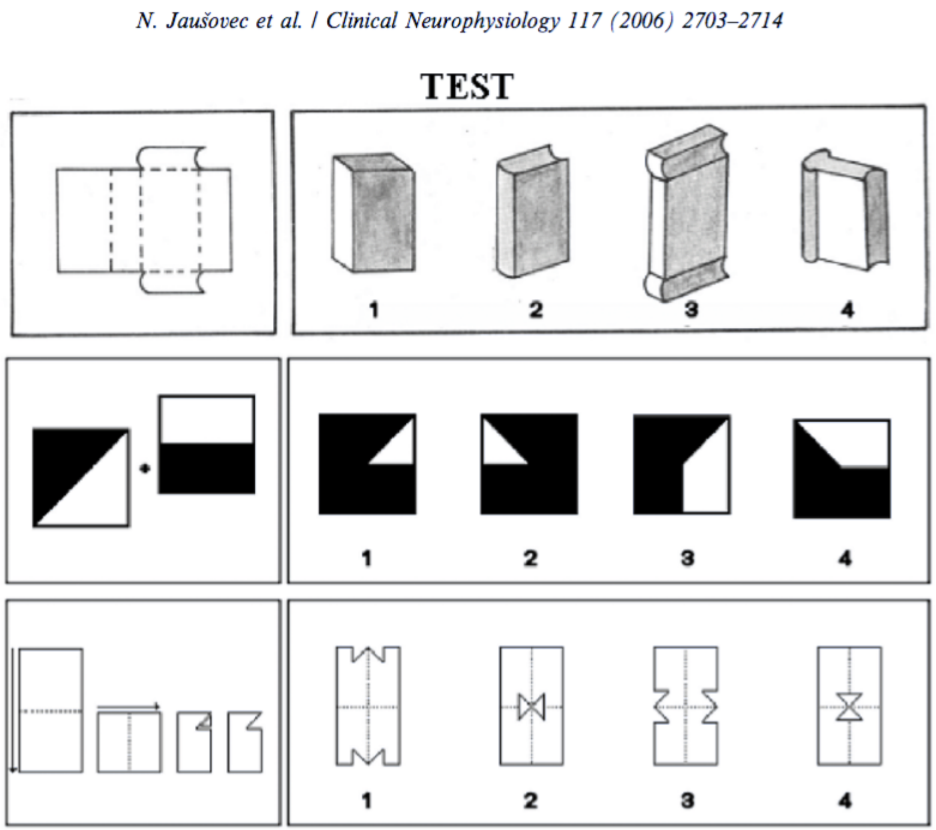
The original paper

- The Authors try to demonstrate a causal relationship between music and spatial-temporal reasoning
- 36 college students participated to the study protocol
- One reasoning test taking from the "Stanford-Binet intelligence scale" was given after 3 conditions: the 1st movement of Mozart piece, relaxation tape, silence
- The subjects participating in the music conditions had scores higher than the other 2 conditions
- The enhancing effect of the music condition was temporal
- The students were not musicians, so it was no possible to discriminate between musician and non-musicians



Standard age scores for each of the three listening conditions.

Example of spatial-temporal task: which of the 4 unfolded pieces of paper in the right frame corresponds to the 1 folded in the left frame?



Spatial temporal reasoning involves maintaining, transforming, and comparing mental mages in space and time, in absence of a physical model

Prelude or requiem for the "Mozart effect"?

- The 1993 study have since proven to be controversial, with some researchers confirming the results, and others being unable to reproduce them

Music and spatial task performance

SIR — There are correlational[1], historical[2] and anecdotal[3] relationships between music cognition and other 'higher brain functions', but no causal relationship has been demonstrated between music cognition and cognitions pertaining to abstract operations such as mathematical or spatial reasoning. We performed an experiment in which students were each given three sets of standard IQ spatial reasoning tasks; each task was preceded by 10 minutes of (1) listening to Mozart's sonata for two pianos in D major, K488; (2) listening to a relaxation tape; or (3) silence. Performance was improved for those tasks immediately following the first condition compared to the second two.

Thirty-six college students participated in all three listening conditions. Immediately following each listening condition, the student's spatial reasoning skills were tested using the Stanford-Binet intelligence scale[4]. The mean standard age scores (SAS) for the three listening conditions are shown in the figure. The music condition yielded a mean SAS of 57.56; the mean SAS for the relaxation condition was 54.61 and the mean score for the silent condition was 54.00. To assess the impact of these scores, we 'translated' them to spatial IQ scores of 119, 111 and 110, respectively. Thus, the IQs of subjects participating in the music condition were 8–9 points above their IQ scores in the other two conditions. A one-factor (listening condition) repeated measures analysis of variance (ANOVA) performed on SAS revealed that subjects performed better on the abstract/spatial reasoning tests after listening to Mozart than after listening to either the relaxation tape or to nothing ($F_{2,35} = 7.08$; $P = 0.002$). The music condition differed significantly from both the relaxation and the silence conditions (Scheffé's $t = 3.41$, $P = 0.002$; $t = 3.67$, $P = 0.0008$, two-tailed, respectively). The relaxation and silence conditions did not differ ($t = 0.795$; $P = 0.432$, two-tailed). Pulse rates were taken before and after each listening condition. A two-factor (listening condition and time of pulse measure) repeated measures ANOVA revealed no interaction or main effects for pulse, thereby excluding arousal as an obvious cause. We found no order effects for either condition presentation or task, nor any experimenter effect.

The enhancing effect of the music condition is temporal, and does not extend beyond the 10–15-minute period during which subjects were engaged in each spatial task. Inclusion of a delay period (as a variable) between the music listening condition and the testing period would allow us quantitatively to determine the presence of a decay constant. It would also be interesting to vary the listening time to optimize the enhancing effect, and to examine whether other measures of general intelligence (verbal reasoning, quantitative reasoning and short-term memory) would be similarly facilitated. Because we used only one musical sample of one composer, various other compositions and musical styles should also be examined. We predict that music lacking complexity or which is repetitive may interfere with, rather than enhance, abstract reasoning. Also, as musicians may process music in a different way from non-musicians, it would be interesting to compare these two groups.

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1. Hassler, M., Birbaumer, N. & Feil, A. Psychol. Music 13, 99–113 (1985).
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3. Cranberg, L. D. & Albert, M. L. in The Exceptional Brain (eds Obler, L. K. & Fein, D.) 156 (Guilford, New York, 1988).
4. Thorndike, R. L., Hagen, E. P. & Sattler, J. M. The Stanford-Binet Scale of Intelligence (Riverside, Chicago, 1986).

Standard age scores for each of the three listening conditions.

Testing procedure. In the music condition, the subject listened to 10 min of the Mozart piece. The relaxation condition required the subject to listen to 10 min of relaxation instructions designed to lower blood pressure. The silence condition required the subject to sit in silence for 10 min. One of three abstract reasoning tests taken from the Stanford-Binet intelligence scale[4] was given after each of the listening conditions. The abstract/spatial reasoning tasks consisted of a pattern analysis test, a multiple-choice matrices test and a multiple-choice paper-folding and cutting test. For our sample, these three tasks correlated at the 0.01 level of significance. We were thus able to treat them as equal measures of abstract reasoning ability.

Scoring. Raw scores were calculated by subtracting the number of items failed from the highest item number administered. These were then converted to SAS using the Stanford-Binet's SAS conversion table of normalized standard scores with a mean set at 50 and a standard deviation of 8. IQ equivalents were calculated by first multiplying each SAS by 3 (the number of subtests required by the Stanford-Binet for calculating IQs). We then used their area score conversion table, designed to have a mean of 100 and a standard deviation of 16, to obtain SAS IQ equivalents.

Scientific Correspondence

Scientific Correspondence is intended to provide a forum in which readers may raise points of a scientific character. They need not arise out of anything published in *Nature*. In any case, priority will be given to letters of fewer than 500 words and five references.

MyoD and c-*fos* expression

SIR — Trouche *et al.*[1] have reported that the down regulation of c-*fos* expression during muscle cell differentiation may result from the binding of the helix-loop-helix (HLH) proteins to a CANNTG motif, or E-box, that occurs within the c-*fos* serum response element (SRE), thereby excluding the binding of the serum response factor (SRF) to this element. We investigated the interaction of HLH proteins with the c-*fos* SRE when molecular clones for E12 were first isolated by screening a phage expression library with the SRE probe[2]. We estimated that the dissociation constant for the myogenin/E12-SRE complex was 10^{-8}–10^{-9} M^{-1} by comparing the relative effectiveness of E-box elements from different genes to compete for binding in the electrophoretic mobility-shift assays. The relatively low affinity of myogenin/E12 for the SRE could result from differences in nucleotide identities at the internal dinucleotide and flanking sequences between the c-*fos* E-box and the consensus HLH binding site[3,4]. By contrast, the dissociation constants for the SRE–SRF or the SRE–SRF/p62[TCF] complexes have been reported to be 5×10^{-10} and $\leq 10^{-11}$ M^{-1}, respectively[5]. Collectively, these data indicate that it is unlikely that HLH proteins alone can significantly compete with SRF for binding to DNA *in vivo*.

It has been reported in other studies that the c-*fos* SRE is either equally active in muscle and non-muscle cells[6], or that it activates muscle-specific expression[7] when situated upstream from a minimal promoter. Further, a comparison of the

Another interesting aspect

The subjects in the original study were 36 college students, so the “Mozart Effect” became *baby centered*?

Maybe, in children whose cortices are highly plastic, the “Mozart Effect” or, in general, music training should produce long-term enhancement of spatial-temporal reasoning

Some experimental observations in neurological disorders

- Exposure in epileptic patients, even in coma, to the K448 showed a (potentially long - term) reduction in neuropathological spiking activity, which characterizes epileptic activity and clinical seizures

- Alzheimer's disease had enhanced short term spatial-temporal reasoning

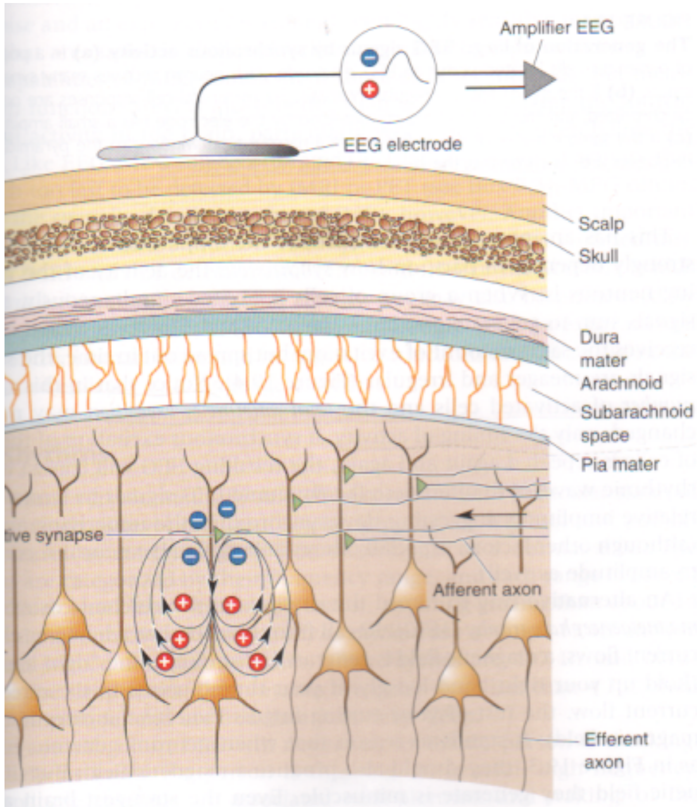
The opinion of Oliver Sacks
("The Man Who Mistook His Wife For a Hat", "Awakenings")

"This so-called Mozart Effect was described, actually, in a very modest way about 15 years ago, and it then got taken up by the media and hyped and exaggerated in a way which was rather embarrassing to the original describers

Real engagement with music, and especially performing music or listening attentively, can make a great deal of difference, especially early in life"

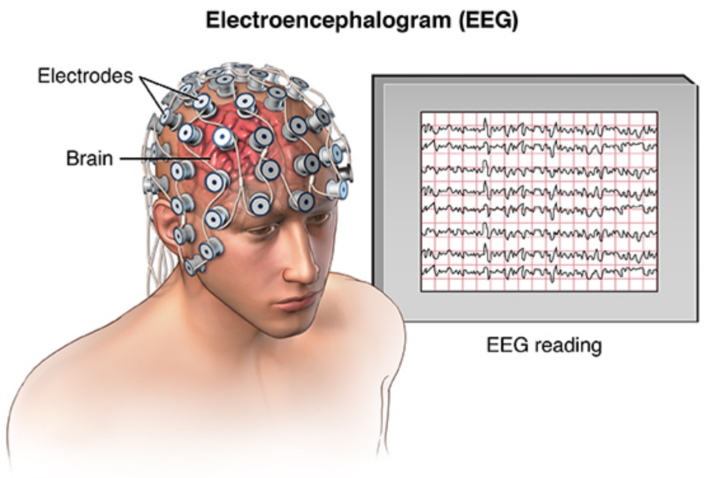
Generally, researchers evaluate the effects of K448 sonata, by analyzing changes in electroencephalographic (EEG) signals of subjects, before, during and after Mozart music

EEG is a recording of the spontaneous electrical activity of the brain from the scalp



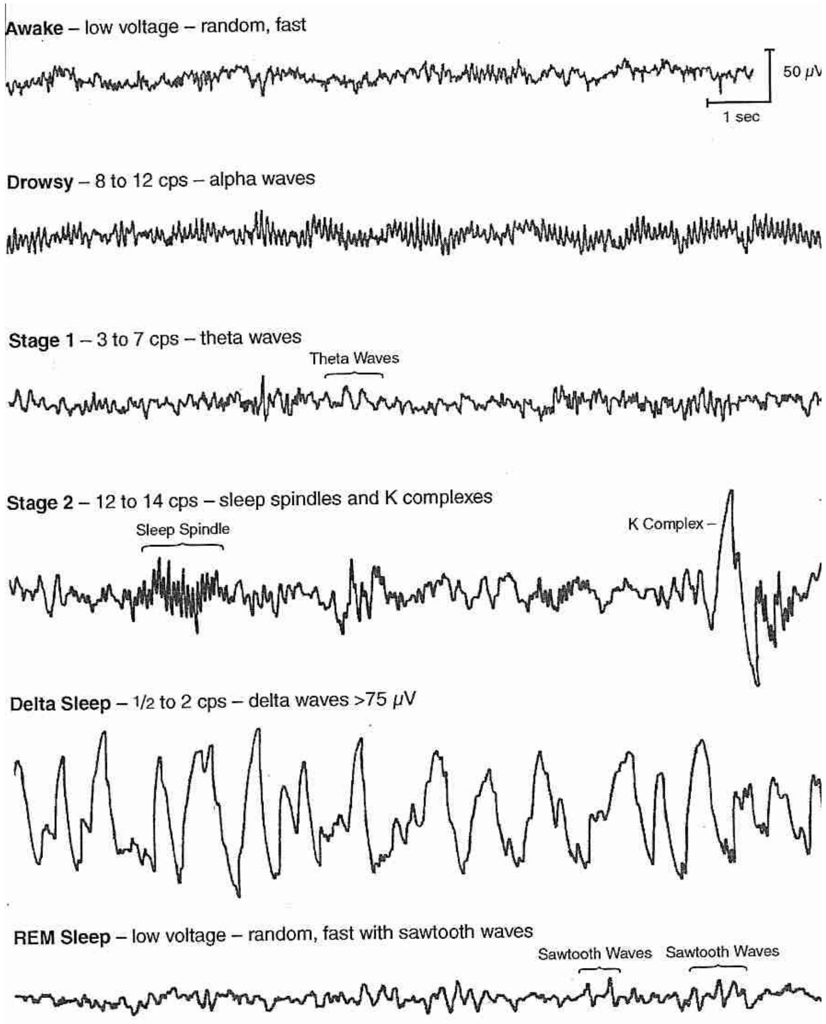
EEG

- The first recording of the electric field of the human brain was made by the German psychiatrist Hans Berger, in 1924
- The internationally standardized 10-20 system is usually employed to record the spontaneous EEG
- In this system 21 electrodes are located on the surface of the scalp
- In addition to these 21 electrodes, intermediate 10% electrode positions are also used



EEG waves

- **Alpha waves: 8 - 12 / 13 Hz**
- **Beta waves: 15 / 18 - 30 Hz**
- Theta waves: 3,5 / 4 - 7,5 / 8 Hz
- Delta waves: 0,75 / 1 - 3 / 4,5 Hz
- **Gamma waves: > 31 Hz**

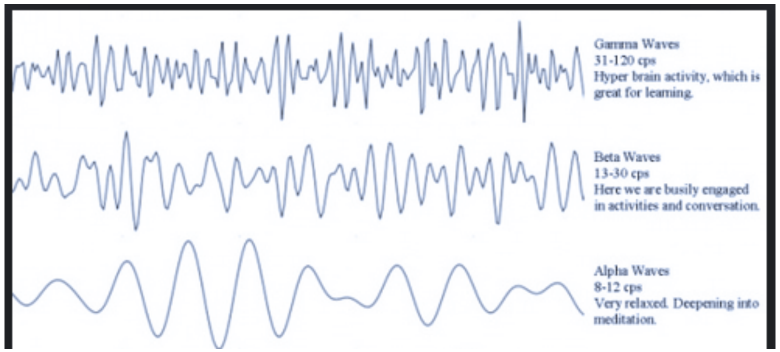


Waves

Alpha waves occur with closed eyes,
relaxation, wandering mind
Alpha waves normally disappear
with attention

Beta waves characterize awake,
normal alert consciousness
Drugs can augment beta waves

For some researchers, **gamma** waves
can be associated to the **music**



“Mozart effect” and EEG

- Some studies on “Mozart effect” have determined the alpha (6 - 12 Hz) and gamma (31 - 49 Hz) waves because these bands are associated to different cognitive function
- The “lower alpha band” is mainly related to attentional tasks demands
- The EEG recordings show that the listening of the K448 activates the same areas of the brain relevant for spatial-temporal tasks

A complementary approach is to characterize the distinctive aspects of the Sonata K448, compared with music of well-known composers (this topic will be developed on Monday)

The trion model and how this model explains the “Mozart Effect”

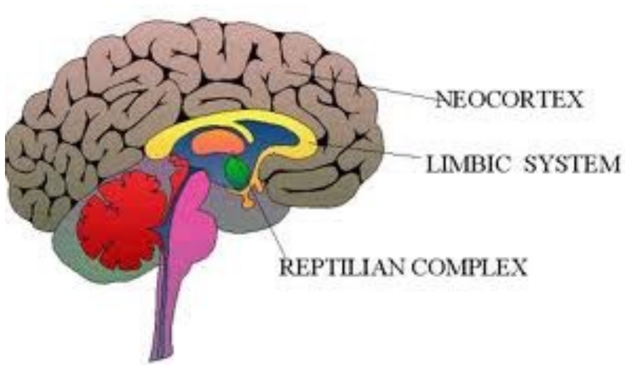
The Shaw’s trion model is a mathematical representation of the columnar organization of the neocortex, proposed in 1957 by Vernon Benjamin Mountcastle (1918, 2015)

Organization of the neocortex

The neocortex of human is a thin, extended, convoluted sheet of tissue with a surface area of ~ 2600 cm^2, and thickness 3 – 4 mm

It contains up to 28 x 10^9 neurons

Cortical neurons are connected by 10^{12} synapses

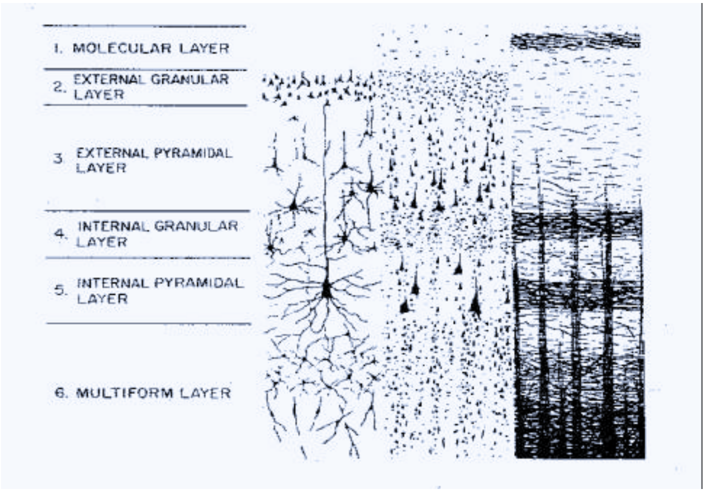


Organization of the neocortex

The basic unit of the neocortex is the **mini-column**, a chain of 80 - 100 neurons extending vertically

Mini-columns contain all the major neural cell types, so they are the **basic elaboration unit** of the neocortex

Cortical columns (about 500 micron in diameter) are formed by many mini-columns bound together by horizontal connections

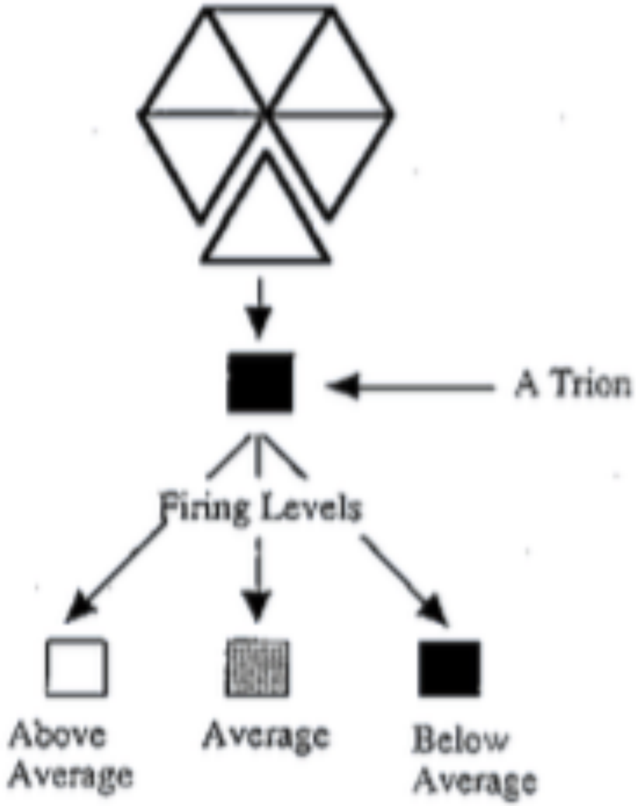


The trion

The **trion** is the representation of the Mountcastle's **mini-column**, i.e. it is a localized group of neurons (30 - 100 neurons)

A **basic network of trions** represents the Mountcastle's **cortical column**

With respect to the HH and MP models, which mainly refer to a single neuron, the trion model is an attempt to abstract the level of individual neurons to a network of neurons



The trion model

A basic network of trions (generally 6 trions) has got inherent repertoire of spatial-temporal firing patterns (MPs = magic patterns)

This network has the capability of being excited and create complex spatial-temporal firing patterns

The creation of such patterns forms the information processing of the neocortex

States of trions

In the network of interconnected neurons, each neuron has 2 possible state and each trion has 3 possible states (or firing levels) S, denoted by **+ 1, 0, and - 1** which represent a firing output **above background, at background, and below background**, respectively

Associated with S, there is a g(S), with g(0) >> g(+ 1) and g(0) >> g(- 1), which takes into account the number of equivalent firing configurations of the trion's internal neuronal constituents

g(0) >> g(+ 1) and g(0) >> g(- 1) give the firing patterns stability

The trion states are updated synchronously in discrete time steps t, which is roughly 30 - 100 ms

Derivation of the state of the system at nt time

S'_j and S''_j are the states of the j^{th} trion at times (n - 1)t and (n - 2)t, respectively

S at nt is updated in a probabilistic way related to the states of the 2 previous discrete time steps

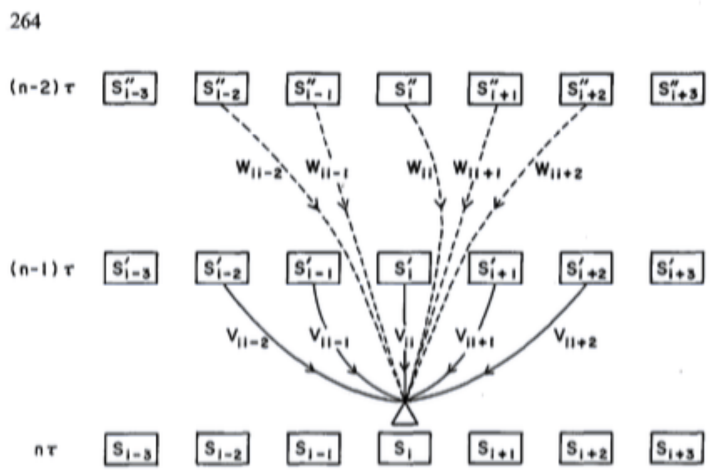
V_{ij} and W_{ij} are the interactions between trions i and j between time nt and times (n - 1)t and (n - 2)t, respectively

V_i^T is the threshold for the i^{th} trion

Derivation of the state of the system at nt time

The picture shows a network of N trions at 3 time steps

Each trion i has connections in one time step (n - 1)t to nt from itself with strength V_{ij} its nearest neighbors V_{ij+1} and V_{ij-1} and its next-nearest neighbors V_{ij+2} and V_{ij-2}



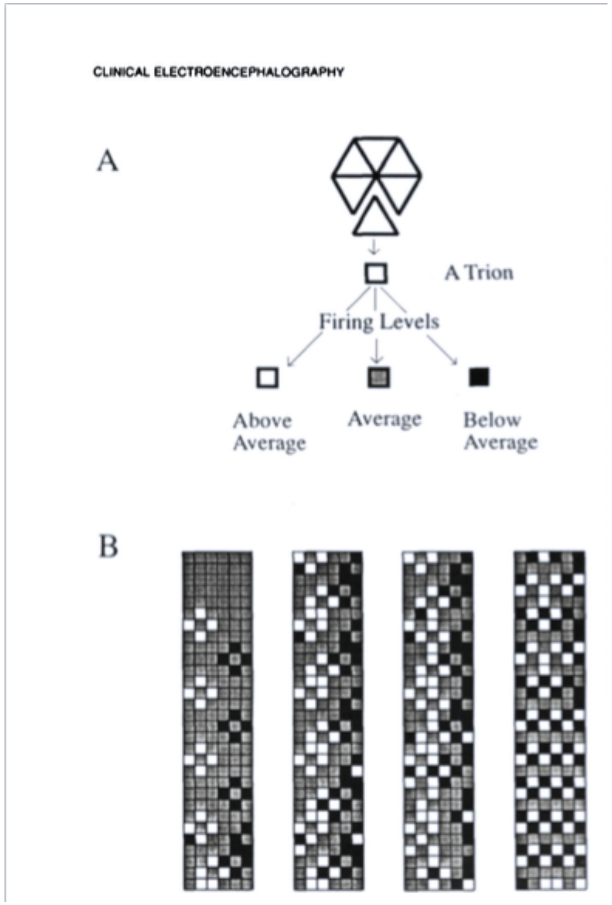
There are similar connections W_{ij} in two time step, (n - 2)t to nt

The probability for the firing level S_i of trion i at nt is determined by the connection strengths, the firing levels S′ and S″ (of the connected trions) at times (n-1)t and (n-2)t, and a parameter B inversely proportional to the noise and/or random fluctuations of synapses in the system:

$$P_i(S) = \frac{g(S)\cdot \exp[B\cdot M_i \cdot S]}{\sum_s g(s)\cdot \exp[B\cdot M_i\cdot s]} \qquad M_i = \sum_j [V_{ij} S'_j + W_{ij} S''_j] - V_i^T,$$

There are cyclic boundary conditions so that trion i = trion i + N

The success of this model rely on the fact that the **basic repertoire of neural firing patterns of the network of trions can be readily enhanced with only small changes in the interaction strengths** (i.e. in the synaptic changes), using learning algorithms



The evolution in time of S

Each row of 6 squares indicates the state of a 6-trion network at a particular discrete point in time, with each individual column indicating the evolution in time of the state of *one* of these trions

The trion model and the “Mozart Effect”

- A small network of trions has inherent neural firing pattern which probabilistically develop in more complex tasks (like spatial-temporal tasks), through small modifications to their connectivity strengths
- Shaw and colleagues suggest that listening to **K448** helps to “organize” the cortical firing patterns specific for spatial-temporal processes, working as an **exercise** aimed to improve connectivity strength
- It is likely that the super-organization of the neocortex resonates with great organization found in Mozart music
- Other factors such as the subjects' age, musical training, and aptitude for the spatial-temporal reasoning may also play a role

- K448 acts as an exercise and priming the inherent repertoire and sequential flow of the cortical firing pattern responsible for higher brain functions
- Music can be used as a “window” into examining higher brain function, since it is the most universally appreciated of the high cognitive processes
- Perhaps the neocortex's response to music is the “Rosetta Stone” for the code or internal language of higher brain function

- Shaw and Rauscher have chosen Mozart since they expect that Mozart (composing by the age of 4) was exploiting the inherent repertoire of spatial-temporal firing patterns in the cortex in a special manner

- As the musical abilities are evident in infants and neonates, music may serve as a “pre-language”, with centers distinct from language centers in the neocortex, which can access inherent cortical spatial-temporal firing patterns and enhance the cortex's ability to accomplish pattern development

- The role of music may be more evident in early ages, and perhaps listening to specific music would give an enhancement of spatial-temporal reasoning

Some unsolved questions

- Are there certain music pieces which are beneficial for specific cognitive processes (not only for spatial-temporal reasoning)?
- Are there certain music pieces which are beneficial for specific pathological conditions?

Some references

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Some exercises

- How can you implement the MP model?
- How can you simplify the HH model if the conductances of sodium and potassium are constants?

https://www.youtube.com/watch?v=v58mf-PB8as