ON COMPUTING THE EIGENVECTORS OF SYMMETRIC TRIDIAGONAL AND SEMISEPARABLE MATRICES

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Abstract

A real symmetric matrix of order n has a full set of orthogonal eigenvectors. The most used approach to compute the spectrum of such matrices reduces first the dense symmetric matrix into a symmetric structured one, i.e., either a tridiagonal matrix [2, 3] or a semiseparable matrix [4]. This step is accomplished in $O(n^3)$ operations. Once the latter symmetric structured matrix is available, its spectrum is computed in an iterative fashion by means of the QR method in $O(n^2)$ operations.

In principle, the whole set of eigenvectors of the latter structured matrix can be computed by means of inverse iteration in $O(n^2)$ operations.

The blemish in this approach is that the computed eigenvectors may not be numerically orthogonal if clusters are present in the spectrum. To enforce orthogonality the Gram-Schmidt procedure is used, requiring $O(n^3)$ operations in the worst case.

In this talk an alternative way of computing the eigenvectors of either symmetric tridiagonal matrices or symmetric semiseparable matrices, known the corresponding eigenvalues, is presented. The algorithm is based on the implicitly shifted QR algorithm. The computed eigenvectors are numerically orthogonal and the complexity of the algorithm is $O(n^2)$ if clusters of small size are present in the spectrum.

References

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