Preconditioned Landweber method for space-invariant image deblurring

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September 13, 2004

The Landweber method is the simplest iterative regularizing algorithm for solving linear ill-posed problems of the form $Af = g$; it is characterized by the recurrence

$$f_{k+1} = f_k + \tau A^*(g - Af_k) \ ,$$

where A^* is the adjoint of the blurring operator A, τ is a parameter controlling the convergence and the initial guess f_0 is usually set to 0. The algorithm enjoys very good regularizing features and is more flexible than the popular Conjugate Gradient (CG) method, but it is often too slow [1].

In the discrete setting of space-invariant image deblurring, we show that an effective acceleration can be provided by the preconditioned variant

$$f_{k+1} = f_k + \tau DA^*(g - Af_k) \ .$$

Here D is a suitable circulant inverse preconditioner of A^*A, which generalizes the approach used in [2] for CG method; in particular, different behaviours can be imposed on the components more related to noise.

A tentative theoretical convergence analysis is given involving eigenvector distributional results, together with a short discussion about the choice of parameters and some preliminary (but promising) numerical experiments.

References

[1] M. Bertero, P. Piana, "Projected Landweber method and preconditioning". *Inverse Problems*, **13** (1997), pp. 441–464.

[2] M. Hanke, J. Nagy, R. Plemmons, "Preconditioned iterative regularization for ill-posed problems". *Numerical Linear Algebra (Kent, OH, 1992)*, de Gruyter, Berlin, 1993, pp. 141–163.

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