

# CURRICULUM VITÆ

April 17, 2016

## Marco Abate

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### 1. Education

June 13, 1985: undergraduate degree in Mathematics, Università di Pisa, with a grade of 110/110 *cum laude*; advisor Prof. E. Vesentini.

July 1985: undergraduate degree in Mathematics of the Scuola Normale Superiore of Pisa.

October 22, 1988: Ph.D. degree in Mathematics, Scuola Normale Superiore, Pisa, advisors Proff. S. Kobayashi, E. Vesentini e J.-P. Vigué, with a grade of 70/70 *cum laude*.

### 2. Employment

June 1, 1989–October 31, 1992: assistant professor in the Department of Mathematics of the Università di Roma Tor Vergata, Italy.

November 1, 1992–October 31, 1993: associate professor of Geometry in the Department of Mathematics of the Università di Roma Tor Vergata, Italy.

November 1, 1993–October 31, 1994: associate professor of Geometry in the Department of Applied Mathematics of the Università di Pisa, Italy.

November 1, 1994–October 31, 1997: full professor of Geometry in the Department of Mathematics of the Università di Ancona, Italy.

March 1996–May 1996: Visiting Professor at the University of California, Berkeley, USA.

November 1, 1997–October 31, 2001: full professor of Geometry in the Department of Mathematics of the Università di Roma Tor Vergata, Italy.

Since November 1, 2001: full professor of Geometry in the Department of Mathematics of the Università di Pisa, Italy.

### 3. Fellowships and awards

January 1986: award *Benedetto Sciarra* for the year 1985, awarded by the Scuola Normale Superiore.

June 1986: Fellowship awarded by the Mathematics Committee of the Italian Consiglio Nazionale delle Ricerche, to support my study in University of Southern California, Los Angeles (September 1986–December 1986) and in University of California, Berkeley (January 1987–June 1987).

October 1987: Research fellowship awarded by the Istituto Nazionale di Alta Matematica (INdAM), to work at the Scuola Normale Superiore of Pisa, renewed for the academical year 1988/89.

May 1989: award of the Fondazione Francesco Severi, Arezzo, for the year 1989.

May 1991: award *Giuseppe Bartolozzi* 1989, awarded by the Unione Matematica Italiana.

### 4. Research grants

October 1998–October 2002: leading scientist of the local unit at the Università di Roma Tor Vergata of the MURST project of relevant national interest (PRIN) *Geometric properties of real and complex manifolds*.

February 2003–October 2006: leading scientist of the local unit at the Università di Pisa of the FIRB project funded by the Italian MIUR *Dynamics and group actions on complex domains and manifolds*.

January 2006–December 2007: Principal investigator for the INdAM project *Local discrete dynamics in one, several, and infinitely many variables*.

March 2015–March 2016: Principal investigator for the Progetto di Ricerca di Ateneo *Connessioni fra dinamica olomorfa, teoria ergodica e logica matematica nei sistemi dinamici*

## 5. Academic responsibilities

- October 2002–October 2005: director of undergraduate studies in Mathematics, Università di Pisa, Italy.
- January 2008–December 2013: member of the Scientific Council and of the Executive Committee of CIME (Centro Italiano Matematico Estivo).
- April 2008–September 2012: national coordinator of the Italian degree courses in Mathematics.
- July 2009–May 2015: member elected of the Scientific Committee of the Unione Matematica Italiana (UMI).
- November 2010–October 2015: head of the Department of Mathematics of the University of Pisa.
- January 2011–December 2015: member of the Council of the Centro di Ricerca Matematica “Ennio de Giorgi”, Pisa.
- December 2011–June 2013: vice president of the Group of Evaluation Experts of the Mathematics/Computer Science area for the Italian research evaluation exercise VQR 2004-2010.
- Since February 2013: member elected for the Mathematics/Computer Science area of the Italian National University Council (CUN).
- Since July 2013: member of the Scientific Council of CISIA (Consorzio Interuniversitario Sistemi Integrati per l’Accesso).
- Since February 2015: coordinator of the 3<sup>rd</sup> Permanent Committee of CUN, on “Politiche per la valutazione, la qualità e l’internazionalizzazione della formazione universitaria (Commissione Didattica)”.
- Since September 2015: president of the Group of Evaluation Experts of the Mathematics/Computer Science area for the Italian research evaluation exercise VQR 2011-2014.
- Referee for: the American *National Science Foundation*, the Italian MIUR (PRIN, FIRB and SIR projects), the Italian agencies CIVR and ANVUR, the Hong Kong *Research Grants Council (RGC)*, the Dutch *NWO*, the French *Agence nationale de la Recherche (ANR)*, the Portuguese *Foundation for Science and Technology (FCT)*, the Romanian *National Council for Scientific Research*, and other agencies.

## 6. Organization of scientific congresses

- May 1999: scientific organizer of the congress *Intensive week on holomorphic dynamics*, Scuola Normale Superiore, Pisa, May 10–14, 1999.
- June 2003: scientific organizer of the congress *Dynamics in Italy*, Scuola Normale Superiore, Pisa, June 25–27, 2003.
- January 2007: scientific organizer of the workshop *Local Holomorphic Dynamics*, Centro di Ricerca Matematica “Ennio de Giorgi”, January 22-26, 2007.
- June 2007: scientific organizer of the special session on *Complex Analysis and Holomorphic Dynamics* in the congress *Joint Meeting UMI-DMV*, Università di Perugia, June 18–22, 2007.
- September 2012: scientific organizer of the INdAM Congress *New trends in holomorphic dynamics*, Cortona, September 3–7, 2012.
- June 2013: scientific organizer of the workshop *Parabolic renormalization*, Centro di Ricerca Matematica “Ennio de Giorgi”, Pisa, June 3–7, 2013.
- March 2015: member of the organizing committee of the EMS Summer School and Workshop *KAWA 6 — Komplex Analysis Weeklong School and Workshop*, Centro di Ricerca Matematica “Ennio de Giorgi”, Pisa, March 22–28, 2015.
- September 2015: member of the scientific committee of the *XX Congresso dell’Unione Matematica Italiana*, Siena, September 7–12, 2015.
- January 2016: member of the scientific committee of the *Geometric aspects of complex dynamics*, Universidade do Porto, Porto, January 11–15, 2016.
- February 2016: scientific organizer of the workshop *Dynamical systems in logic, complex analysis and ergodic theory*, Pisa, February 8–9, 2016.

## 7. Organization of popularizing and outreach congresses

- April 2005: organizer of the congress *Perché Nobel? 2004*, University of Pisa, first two weeks of April 2005.
- March 2006: organizer of the congress *Perché Nobel? 2005*, University of Pisa, last week of March and first week of April 2006.
- March 2007: organizer of the congress *Perché Nobel? 2006*, University of Pisa, last two weeks of March 2007.
- March 2008: organizer of the congress *Perché Nobel? 2007*, University of Pisa, last week of March and first week of April 2008.
- March 2009: organizer of the congress *Perché Nobel? 2008*, University of Pisa, last two weeks of March 2009.
- March 2010: organizer of the congress *Perché Nobel? 2009*, University of Pisa, first two weeks of March 2010.
- March 2010: member of the scientific committee of the congress *Matematica e cultura 2010*, University Ca' Foscari, Venezia, March 26–28, 2010.
- March 2011: organizer of the congress *Perché Nobel? 2010*, University of Pisa, first two weeks of March 2011.
- March 2011: member of the scientific committee of the congress *Matematica e cultura 2011*, Istituto Universitario di Architettura, Venezia, March 25–27, 2011.
- March 2012: member of the scientific committee of the congress *Matematica e cultura 2012*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, March 30–April 1, 2012.
- March 2013: member of the scientific committee of the congress *Matematica e cultura 2013*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, March 22–24, 2013.
- March 2014: member of the scientific committee of the congress *Matematica e cultura 2014*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, March 28–30, 2014.
- March 2015: member of the scientific committee of the congress *Matematica e cultura 2015*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, March 27–29, 2015.

## 8. Editorial responsibilities

- Since July 2009: *associated editor* of the *Journal of Geometric Analysis*.
- Since November 2009: *editor* of the journal *Bollettino dell'Unione Matematica Italiana*.
- October 2012–December 2015: member of the *advisory board* of the web journal *MaddMaths*.
- Since July 2013: *associated editor* of the journal *Complex Analysis and its Synergies*.
- Since March 2014: *editor-in-chief* of the book series *Quaderni della Settimana Matematica*.
- Referee for: *Acta Math.*; *Adv. Geom.*; *Adv. Math.*; *Ann. Acad. Sci. Fenn.*; *Ann. Scuola Norm. Sup.*; *Bull. Canad. Math. Soc.*; *Boll. Mat. Univ. Padova*; *C. Eur. J. Math.*; *Complex Var. Th. Appl.*; *Complex Var. Elliptic Eq.*; *Compos. Math.*; *Duke Math. J.*; *Houston Math. J.*; *Int. J. Math. Math. Sci.*; *Int. Math. Res. Notices*; *Int. Eq. Oper. Theory*; *J. Austr. Math. Soc.*; *J. Diff. Geom.*; *J. Europ. Math. Soc.*; *J. Geom. Anal.*; *J. Math. Anal. Appl.*; *Math. Ann.*; *Math. Res. Letter*; *Math. Z.*; *Mich. Math. J.*; *Missouri Math. J.*; *Pac. J. Math.*; *Proc. Amer. Mat. Soc.*; *Publ. Math. Debr.*; *Rend. Accad. Naz. Lincei*; *Rend. Mat.*; *Trans. Am. Math. Soc.* and other journals.

## 9. Graduate students

- Marzia Rivi: *Local behavior of discrete dynamical systems*, Università di Firenze, Ph.D. Thesis, A.A. 1998/99.
- Laura Molino: *Local holomorphic dynamics at a non-hyperbolic fixed point*, Università di Roma Tor Vergata, Ph.D. Thesis, A.A. 2003/04.
- Jasmin Raissy: *Geometrical methods in the normalization of germs of biholomorphisms*, Università di Pisa, Ph.D. Thesis, A.A. 2009/10.
- Tiziano Casavecchia: *Rigidity of holomorphic generators of one-parameter semigroups and a non-autonomous Denjoy-Wolff theorem*, Università di Pisa, Ph.D. thesis, A.A. 2009/10.
- Matteo Ruggiero: *The valuative tree, rigid germs and Kato varieties*, Scuola Normale Superiore, Pisa, Ph.D. Thesis, A.A. 2010/11.

- Isaia Nisoli: *A general approach to Lehmann-Suwa-Khanedani index theorems: partial holomorphic connections and extensions of foliations*, Università di Pisa, Ph.D. Thesis, A.A. 2010/11.

Furthermore, I have been the advisor for 23 undergraduate theses, in the Università di Roma Tor Vergata and in the Università di Pisa.

## 10. Graduate and post-graduate teaching

- April 1991: *Iteration theory of holomorphic maps*, short graduate course, Ruhr-Universität, Bochum, Germany.
- February 1994–May 1994: *Iteration theory in one complex variable*, one semester graduate course, Università di Pisa, Italy.
- July 1996: *Complex Dynamics*, post-graduate course, Scuola Matematica Interuniversitaria (SMI), Cortona, Italy.
- January 1998–March 1998: *Complex analysis*, one-semester graduate course, Istituto Nazionale di Alta Matematica (INdAM), Roma, Italy.
- October 2000–January 2001: *Introduction to discrete hyperbolic dynamical systems*, one semester graduate course, Università di Pisa, Italy.
- July 2002: *Angular derivatives in several complex variables*, short post-graduate course, in the CIME school *Real methods in complex and CR geometry*, Martina Franca, Italy.
- January 2005: *Local holomorphic dynamics*, short graduate course, Université Paul Sabatier, Toulouse, France.
- September 2007: *An introduction to local discrete holomorphic dynamics*, short post-graduate course in the *III International Course of Mathematical Analysis in Andalucía*, Universidad Internacional de Andalucía, La Rábida, Spain.
- December 2007: *An introduction to local discrete holomorphic dynamics*, short graduate course, University of Cyprus.
- January 2008: *An overview of local dynamics in several complex variables*, short graduate course, in the Graduate School *Complex Dynamics*, University of Liverpool, Great Britain.
- July 2008: *Discrete holomorphic local dynamical systems*, short post-graduate course, in the CIME school *Holomorphic dynamical systems*, Cetraro, Italy.
- November 2010: *An introduction to holomorphic dynamics*, short graduate course, Al-Fateh University, Tripoli, Libia.
- May 2011: *An introduction to local holomorphic dynamics in several variables*, short post-graduate course, in the school *Topics in complex dynamics*, IMUB, Barcelona.
- December 2013: *An introduction to holomorphic dynamics*, short graduate course, Tribhuvan University, Kathmandu, Nepal.
- May 2015: *Invariant distances in complex analysis and dynamics*, short graduate course, in the school *Aspects mtriques et dynamiques en analyse complexe*, Université de Lille, France.
- November 2015: *An introduction to holomorphic dynamics*, short graduate course, The Institute of Mathematical and Physical Sciences (IMSP), Porto-Novo, Benin.

And of course I have taught dozens of undergraduate courses in the Università di Roma Tor Vergata, Ancona and Pisa.

## 11. Research talks in universities and congresses

- 1986: Scuola Normale Superiore, Pisa, Italy, 2 talks.
- 1987: University of California, Berkeley, USA.  
Scuola Normale Superiore, Pisa, Italy.
- 1988: invited talk, *Giornate di Geometria Analitica e Analisi Complessa*, Rocca di Papa, Italy.  
Università della Calabria, Rende, Italy.  
Università di Parma, Italy.

- SISSA, Trieste, Italy.  
invited talk, *Complex Analysis and Geometry VII*, CIRM, Trento, Italy.
- 1989: invited talk, *Summer Research Institute on Several Complex Variables and Geometry*, American Mathematical Society, Santa Cruz, USA.
- 1990: Università di Roma Tor Vergata, Italy, 3 talks.  
Università di Pisa, Italy.  
Washington University, St. Louis, USA.  
Indiana University, Bloomington, USA.  
Purdue University, West Lafayette, USA.  
invited talk, *Minicorso sulla Trasformata di Radon*, Università della Calabria, Rende, Italy.
- 1991: Università di Roma Tor Vergata, Italy, 3 talks.  
Bremen Universität, Bremen, Germany.  
invited talk, *Congresso dell'Unione Matematica Italiana*, Catania, Italy.
- 1992: Università di Roma Tor Vergata, Italy.  
Institute for Advanced Study, Princeton, USA.  
Università di Bologna, Italy.  
Scuola Normale Superiore, Pisa, Italy.  
Università di Trieste, Italy.
- 1993: University of Michigan, Ann Arbor, USA.  
invited talk, *Invariant Metrics and Related Questions in Differential Geometry and Complex Analysis*, Cortona, Italy.  
Università di Pisa, Italy.
- 1994: Università di Ancona, Italy.  
invited talk, *Complex dynamics in higher dimensions, NSF-CNMS Regional Conference*, State University of New York, Albany, USA.  
invited talk, *Third Analysis Colloquium*, University of Bern, Switzerland.
- 1995: Università di Ancona, Italy.  
Università di Firenze, Italy.  
invited talk, *Summer Research Conference on Finsler Geometry*, American Mathematical Society, University of Washington, Seattle, USA.
- 1996: University of California, Berkeley, USA.
- 1997: invited talk, *International Conference on Several Complex Variables*, Postech University, Pohang, South Korea.  
invited talk, *Convegno nazionale del GNSAGA*, Università di Perugia, Italy.
- 1998: Università di Roma Tor Vergata, Italy, 2 talks.  
invited talk, *JAMI Conference on Meromorphic Mappings and Intrinsic Metrics in Complex Geometry*, John Hopkins University, Baltimore, USA.  
Scuola Normale Superiore, Pisa, Italy, 2 talks.
- 1999: Scuola Normale Superiore, Pisa, Italy.  
talk, *Reelle Methoden der Komplexen Analysis*, Mathematisches Forschungsinstitut Oberwolfach, Germany.  
Tulane University, New Orleans.  
invited talk, *Complex dynamics*, University of Arkansas, Fayetteville, USA.  
University of Auckland, New Zealand.  
Università di Roma Tor Vergata, Italy.  
invited talk, *Hayama Symposium on Several Complex Variables*, Tokyo, Japan.
- 2000: Università di Pisa, Italy.  
Università di Roma La Sapienza, Italy.  
invited talk, *Biholomorphic mappings*, American Institute of Mathematics, Palo Alto, USA.  
invited talk, *Complex analysis and analytic geometry*, Cortona, Italy.

- University College, Dublin, Ireland.
- 2001: invited talk, *Fixed point theory and its applications*, Haifa, Israel.  
 invited talk, *Systèmes dynamiques en plusieurs variables complexes*, CIRM, Luminy, France.
- 2002: Scuola Normale Superiore, Pisa, Italy.  
 invited talk, *Complex dynamics conference*, University of Michigan, Ann Arbor, Italy.  
 Centro di ricerca matematica “Ennio de Giorgi”, Pisa, Italy.
- 2003: invited talk, *Thirteenth seminar on analysis and its applications*, University of Isfahan, Iran.  
 invited talk, *Dinamica in Italia*, Scuola Normale Superiore, Pisa, Italy.  
 invited talk, *Complex dynamics in higher dimension*, RIMS, Kyoto, Japan.
- 2004: invited talk, *Fifth International conference on dynamical systems and differential equations*, AIMS, California Polytechnic University, Pomona, USA.  
 invited talk, *2004 Beijing International Conference on Several Complex Variables*, Capital Normal University, Beijing, China.  
 invited talk, *Giornata di analisi complessa*, Università di Firenze, Italy.
- 2005: invited talk, *Journées complexes du sud*, Moissac, France.  
 invited talk, *Complex Analysis and Geometry XVII*, CIRM, Trento, Italy.  
 Jagellonian University, Kraków, Poland, 2 talks.  
 Centro di ricerca matematica “Ennio de Giorgi”, Pisa, Italy.  
 University of Michigan, Ann Arbor, USA.
- 2006: University of Uppsala, Sweden.  
 University of Ljubljana, Slovenia.  
 invited talk, *Symposium in complex analysis, Slovenia 2006*, Kranjska Gora, Slovenia.  
 invited talk, *Mathematics and its applications, Joint Meeting UMI-SMF*, Torino, Italy.
- 2007: Université Paul Sabatier, Toulouse, France.  
 Kyoto University, Japan.  
 invited talk, *6th Congress of Romanian Mathematicians*, University of Bucarest, Romania.  
 Università di Roma Tor Vergata, Italy.  
 Università di Roma La Sapienza, Italy.
- 2008: Institut Mittag-Leffler, Stockholm, Sweden.  
 invited talk, *Algebraic Geometry, D-modules, Foliations and their interactions*, University of Buenos Aires, Argentina.  
 invited talk, *Workshop on Holomorphic iteration, semigroups and Loewner chains*, INdAM, Roma, Italy.  
 invited talk, *Workshop on Geometry of projective varieties*, INdAM, Roma, Italy.
- 2009: Institut de Matemàtica, Universitat de Barcelona, Spain.  
 invited talk, *Giornate di geometria 2*, Università di Pavia, Italy.  
 invited talk, *Multivariable complex dynamics*, BIRS, Banff, Canada.  
 Centro di Ricerca Matematica “Ennio de Giorgi”, Pisa, Italy, 2 talks.  
 Université de Paris-Sud, Orsay, France.  
 invited talk, *7th ISAAC Congress*, Imperial College, London, Great Britain.  
 invited talk, *Midwest several complex variables conference*, Purdue University, West Lafayette, USA.  
 Università di Pisa, Italy.  
 ETH, Zurich, Switzerland.
- 2010: invited talk, *Komplex Analysis Winter Workshop*, Albi, France.  
 Korteweg-de Vries Institute for Mathematics, University of Amsterdam, Netherlands.  
 Università di Milano Bicocca, Italy.  
 invited talk, *ICM 2010 Satellite conference on Various aspects of dynamical systems*, The Maharaja Sayajirao University of Baroda, Vadodara, India,  
 invited talk, *AMS Sectional meeting Fall 2010*, Syracuse University, Syracuse, USA.  
 Rutgers University, New Brunswick, USA.

- 38th William J. Spencer Lecture, Department of Mathematics, Kansas State University, Manhattan, USA.  
 invited talk, *Tenth Prairie Analysis Seminar*, University of Kansas, Lawrence, USA.
- 2011: Institut de Matemàtica, Universitat de Barcelona, Spain.  
 invited talk, *Workshop in Several Complex Variables*, Korteweg-de Vries Institute for Mathematics, Amsterdam, Netherlands.
- 2012: invited talk, *Geometria in Bicocca*, Università di Milano Bicocca, Italy.  
 invited talk, *58th workshop: Variational Analysis and Applications*, Centro e Fondazione Ettore Majorana per la Cultura Scientifica, Erice, Italy.  
 invited talk, *Interactions between continuous and discrete holomorphic dynamical systems*, BIRS, Banff, Canada.  
 invited talk, *9th Korean Conference on Several Complex Variables*, GyeongJu, South Korea.  
 Université Paul Sabatier, Toulouse, France.
- 2013: invited talk, *2013 Joint Mathematics Meetings*, American Mathematical Society, San Diego, USA.  
 invited talk, *KAWA IV*, Albi, France.  
 invited talk, *Varietà reali e complesse: geometria, topologia e analisi armonica*, Scuola Normale Superiore, Pisa, Italy.  
 Centro di Ricerca Matematica “Ennio de Giorgi”, Pisa, Italy.  
 invited talk, *Complex analysis and Dynamical systems VI*, ORT Braude College, Nahariya, Israel.  
 invited talk, *NORDAN 2013*, Svolve, Norway.  
 invited talk, *Workshop in real and complex dynamics*, ICTP, Trieste.  
 invited talk, *Parabolic renormalization*, Centro di Ricerca Matematica “Ennio de Giorgi”, Pisa, Italy.  
 invited talk, *Workshop in function theory and complex variables*, Şirince, Turkey.  
 invited talk, *Complex analysis and related topics*, Ivan Franko National University, L’viv, Ukraine.
- 2014: Jyväskylä University, Jyväskylä, Finland.  
 invited talk, *TSIMF Symposium on Complex Analysis and Complex Dynamics*, Tsinghua Sanya International Mathematics Forum, Sanya, China.  
 invited talk, *First Joint International Meeting RSME-SCM-SEMA-SIMAI-UMI*, University of the Basque County, Bilbao, Spain.  
 Scuola Normale Superiore, Pisa, Italy.  
 invited talk, *KSCV Symposium X: International conference on Complex Analysis and Geometry (ICM 2014 Satellite Conference)*, GyeongJu, South Korea.  
 invited talk, *Complex geometry, analysis and foliations*, ICTP, Trieste.
- 2015: IMPA, Rio de Janeiro, Brazil.  
 invited talk, *Conference on Dynamical Systems*, ICTP, Trieste.  
 Stefan Banach Institute, Warsaw, Poland.
- 2016: invited talk, *Geometric aspects of complex dynamics*, Universidade do Porto, Porto, Portugal.  
 invited talk, *Dynamical systems in logic, complex analysis and ergodic theory*, Università di Pisa, Italy.

## 12. Popularizing talks

- 2003: *Scrivere Matematica nel fumetto*, invited talk, *Matematica e cultura VII*, Università di Ca’ Foscari, Venezia, Italy.
- 2005: *Matematica e fumetti*, Scuola Normale Superiore, Pisa, Italy.  
*Matematica e algoritmi*, Biblioteche di Roma, Italy.  
*Évariste et Héloïse, ovvero come scrivere una storia ispirata a Galois?*, invited talk, *Matematica e cultura IX*, Università di Ca’ Foscari, Venezia, Italy.  
*Perché Michael Atiyah e Isadore Singer hanno ricevuto il Premio Abel 2004?*, invited talk, *Perché Nobel? 2004*, Università di Pisa, Italy.  
*Évariste et Héloïse, ovvero come scrivere una storia ispirata a Galois?*, SISSA, Trieste, Italy.  
*Évariste et Héloïse, ovvero come scrivere una storia ispirata a Galois?*, invited talk, *Venezia a Roma*, Università di Roma La Sapienza, Italy.

*Leonardo Pisano, il Fibonacci*, Centro Servizi Montipisani, Calci, Italy.

2006: *Il girasole di Fibonacci*, invited talk, *Matematica e cultura X*, Università di Ca' Foscari, Venezia, Italy.

2007: *L'autobiografia riluttante di G.H. Hardy*, invited talk, *Matematica e cultura XI*, Università di Ca' Foscari, Venezia, Italy.

*Perché Lennart Carleson ha ricevuto il Premio Abel 2006?*, invited talk, *Perché Nobel? 2006*, Università di Pisa, Italy.

*Il girasole di Fibonacci*, Università di Bologna, Italy.

2008: *Il linguaggio della Matematica*, Università di Roma Tor Vergata, Italy.

*L'autobiografia riluttante di G.H. Hardy*, Centro di Ricerca Matematica "Ennio de Giorgi", Pisa, Italy.

2009: *Prezzi nel caos*, invited talk, *Matematica e cultura XII*, Università di Ca' Foscari, Venezia, Italy.

*Prezzi nel caos* Università di Trieste, Italy.

2010: *Quando il cielo ci cade sulla testa*, invited talk, *Matematica e cultura 2010*, Università di Ca' Foscari, Venezia, Italy.

2011: *I nodi di Lorenz*, invited talk, *Matematica e cultura 2011*, Istituto Universitario di Architettura di Venezia, Italy.

*Quando il cielo ci cade sulla testa*, Scuola di orientamento preuniversitario, Quarrata, Italy.

2012: *Perché John Milnor ha vinto il Premio Abel 2011?*, invited talk, *Perché Nobel? 2012*, Università di Pisa, Italy.

*Sfere esotiche e John Milnor*, invited talk, *Matematica e cultura 2012*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, Italy.

*Curve annodate e sfere esotiche*, Olimpiadi della Matematica, Cesenatico, Italy.

2013: *Alla ricerca delle radici perdute*, invited talk, *Matematica e cultura 2013*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, Italy.

2014: *Arte frattale*, invited talk, *Matematica e cultura 2014*, Istituto Veneto di Scienze, Lettere e Arti, Venezia, Italy.

### 13. Main research achievements

In this section I shall briefly list my main research achievements; the next section contains a more detailed description of my research work. The numbers in square brackets refer to the list of publications.

#### 13.1. Holomorphic dynamical systems.

- The complete description of the dynamics of holomorphic self-maps of complex taut manifolds and of convex and pseudoconvex domains ([1.6, 1.11, 3.1, 1.43], the latter in collaboration with J. Raissy).
- The development of a general procedure for reducing the study of the local dynamics of a germ with non-diagonalizable differential to the study of the local dynamics of a related germ with diagonalizable differential ([1.24]).
- A Leau-Fatou flower theorem for bidimensional holomorphic germs tangent to the identity ([1.25]).
- The study of the dynamics of homogeneous vector fields, providing in particular the first example of description of the local dynamics in a full neighborhood of the fixed point for a substantial class of examples of germs tangent to the identity ([1.35], in collaboration with F. Tovena).

#### 13.2. Complex differential geometry.

- The development of the global differential geometry of complex Finsler manifolds, and in particular the classification of complete Finsler manifolds with constant nonnegative holomorphic sectional curvature, and the study of the geometry of complete Finsler manifolds with constant negative holomorphic sectional curvature ([1.18, 1.20, 3.2], in collaboration with G. Patrizio).
- The discovery of several index theorems for holomorphic self-maps having positive dimensional fixed point set, and the development of a general framework encompassing in a unified theory index theorems for both holomorphic foliations and holomorphic self-maps ([1.29, 1.31], in collaboration with F. Bracci and F. Tovena).
- The study of the asymptotic behavior of geodesics for meromorphic connections on Riemann surfaces ([1.35], in collaboration with F. Tovena, and [1.45], in collaboration with F. Bianchi).



### 13.3. Geometric function theory and geometric analysis of several complex variables.

- The study of the boundary behavior of the Kobayashi distance and of complex geodesics in strongly pseudoconvex domains ([1.3, 3.1]).
- The generalization to domains in several complex variables of the classical one-dimensional Lindelöf principle and Julia-Wolff-Carathéodory theorem about the boundary behavior of holomorphic maps and their derivatives ([1.9, 2.3, 1.23, 1.27, 2.13], the latter in collaboration with R. Tauraso).
- The geometric characterization of Carleson measures for weighted Bergman spaces in strongly pseudoconvex domains, with application to the study of mapping properties of Toeplitz operators ([1.36, 1.39], in collaboration with A. Saracco and the latter also with J. Raissy).

## 14. Research work

My research work has mainly been in three, closely interrelated, areas: Holomorphic dynamical systems, Complex differential geometry, and Geometric function theory and geometric analysis of several complex variables. The numbers in square brackets still refer to the list of publications.

### 14.1. Holomorphic dynamical systems.

*14.1.1. Global discrete dynamical systems on taut manifolds.* From the point of view of the study of global holomorphic dynamical systems, complex manifolds naturally separates in two classes: Kobayashi hyperbolic (or, more precisely, taut) manifolds (e.g., bounded domains in  $\mathbb{C}^n$ ), and not Kobayashi hyperbolic manifolds (e.g.,  $\mathbb{C}^n$  or  $\mathbb{P}^n(\mathbb{C})$ ). The first part of my work in this area consists in a complete description of the dynamics of holomorphic self-maps of taut manifolds (a complex manifold  $X$  is taut if the family of holomorphic maps from the unit disk  $\Delta \subset \mathbb{C}$  into  $X$  is a normal family in the sense of Montel); the results obtained and the methods introduced have spawned a large body of literature, giving rise to a field of research which is still very active today.

Paper [1.6] is devoted to the study of holomorphic dynamical systems in smoothly bounded strongly convex domains. The main result is the generalization to this setting of the classical one-variable Wolff-Denjoy theorem: the sequence of iterates of a fixed point free holomorphic self-map a smoothly bounded strongly convex domain converges, uniformly on compact subsets, to a point in the boundary, the Wolff point of the map. A key rôle in the proof is played by the invariant Kobayashi distance (see 14.3.1 below), and in particular by the boundary estimates proved in [1.3]. In [2.2] the same problem is studied in smoothly bounded weakly convex domains, where the sequence of iterates might converge to a non-constant map. More than 20 years later, in [1.43], written in collaboration with J. Raissy, we generalized these results to bounded convex domains not necessarily smooth.

Papers [2.1] and [1.11] contain the complete description of the asymptotic behavior of the sequence of iterates of a holomorphic self-map of a taut manifold  $X$ ; in particular, a necessary and sufficient condition is found for the convergence of the sequence of iterates to a holomorphic self-map of  $X$ , and it is fully characterized the set of limits of subsequences of iterates.

Paper [1.11] moreover studies the problem of determining when a sequence of iterates of a holomorphic self-map of a taut manifold is diverging on compact sets (i.e., it converges to the boundary); it turns out that, under suitable topological hypotheses on the manifold, this happens if and only if the map has no periodic points. Furthermore, in [1.11] results of [1.6, 2.2] are generalized to smooth strongly pseudoconvex domains and to some domains of finite type.

Related to [1.11] is paper [1.15], written in collaboration with P. Heinzner, where it is described a taut topologically contractible bounded pseudoconvex domain in  $\mathbb{C}^8$ , strongly pseudoconvex everywhere but in one point, where a finite cyclic group acts holomorphically without fixed points. In particular, this shows that the characterization of holomorphic self-maps of topologically trivial taut manifolds whose sequence of iterates is not compactly divergent cannot be improved: there are periodic holomorphic self-maps of topologically trivial taut manifolds without fixed points.

In [1.17] it is shown that the asymptotic behavior of the iterates of a family of holomorphic self-maps of a taut manifold depending holomorphically on a parameter is essentially the same for all maps in the family.

The book [3.1] deserves a special mention. It contains a complete and self-sufficient presentation of the theory of holomorphic dynamical systems on hyperbolic Riemann surfaces in one variable, and on taut manifolds in several variables, updated up to 1989 (when the book was published). Furthermore, it contains an extensive introduction to the theory of invariant metrics and distances in several variables (see 14.3.1),

and an exposition of related arguments (Lempert's theory of complex geodesics — see 14.3.1 —, Lindelöf principles and angular derivatives in one and several complex variables — see 14.3.2 —, common fixed points of commuting maps — see 14.3.4 — and others), most of them never before published in book form. Furthermore, the book also contains an extensive bibliography and accurate historical notes. After more than 20 years from its publication, this monograph still is a fundamental and much quoted reference work for these subjects.

[1.37], written in collaboration with J. Raissy, studies inverse orbits of holomorphic self-maps of strongly convex domains, generalizing results obtained by Poggi-Corradini and Bracci in dimension one, and by Ostapyuk in the unit ball of  $\mathbb{C}^n$ .

*14.1.2. Global continuous dynamical systems on taut manifolds.* Among the possible global continuous holomorphic dynamical systems, I have studied one-parameter semigroups of holomorphic self-maps of a complex manifold  $X$ , that is continuous homomorphisms from  $\mathbb{R}^+$  into the topological semigroup of holomorphic self-maps of  $X$ .

In [1.7] I adapted the techniques introduced in [1.6] to study one-parameter semigroups of holomorphic self-maps of smoothly bounded strongly convex domains, characterizing in particular the converging semigroups.

Paper [1.12] introduces the basic notion of infinitesimal generator of a one-parameter semigroup of holomorphic self-maps, and gives a characterization of the vector fields that can be infinitesimal generators in complete Kobayashi hyperbolic manifolds.

In [2.22], written in collaboration with F. Bracci, we show that if  $p_0 \in \partial D$  is a boundary regular fixed point for one map of a one-parameter semigroup of holomorphic self-maps of a bounded strongly convex domain  $D \subset \subset \mathbb{C}^n$  then it is a boundary regular fixed point for all maps of the semigroup. The proof is based on results contained in [1.37].

In [1.44], written in collaboration with J. Raissy, we study the boundary behavior of infinitesimal generators of one-parameter semigroups in the unit ball of  $\mathbb{C}^n$ , obtaining a Julia-Wolff-Carathéodory theorem (see 14.3.2) extending and making more precise previous results by Bracci and Shoikhet.

[1.47] is a survey paper where, among other things, I present an updated description of this theory.

*14.1.3. Local holomorphic dynamics.* A local discrete holomorphic dynamical system is given by a germ of holomorphic self-map  $f$  of a complex manifold  $X$  defined in a neighborhood  $U \subset X$  of a fixed point  $p_0 \in U$ ; studying its dynamics means studying the topological, geometrical and dynamical structure of the set of points  $p \in U$  whose forward  $f$ -orbit is completely contained in  $U$ .

Thanks to the work of Königs, Böttcher, Leau, Fatou, Écalle, Yoccoz, Perez-Marco and many others, the local dynamics in one complex variable is more or less completely understood. On the other hand, with the exception of the hyperbolic case (when the differential of  $f$  at the fixed point is invertible and has no eigenvalues of modulus one) where one can apply the stable manifold theorem, in several complex variables many basic problems are still completely open.

A particular interesting case is when all the eigenvalue of the differential of  $f$  at the fixed point  $p$  are equal to 1. In the '80's, Écalle, Hakim and others have studied the tangent to the identity case, when  $df_p = \text{id}$ , giving sufficient conditions (valid for generic maps) for the existence of parabolic curves, i.e., holomorphic embedded  $f$ -invariant one-dimensional curves having  $p$  in their boundary and attracted by  $p$  under the action of  $f$ ; parabolic curves are the natural generalizations of the attracting petals in the classical Leau-Fatou flower theorem.

In [1.24] I have considered the case when  $df_p$  is not diagonalizable, introducing a general construction associating to the germ  $f$  a germ  $\tilde{f}$  in a new complex manifold  $\tilde{X}$  obtained by  $X$  via a series of blow-ups of suitable submanifolds, at a point  $\tilde{p} \in \tilde{X}$  so that  $\tilde{f}$  is semi-conjugated to  $f$  and  $d\tilde{f}_{\tilde{p}}$  is diagonalizable. When all eigenvalues of  $df_p$  are equal to 1 one obtains  $d\tilde{f}_{\tilde{p}} = \text{id}$ , and hence one can get a number of interesting results on the dynamics of  $f$  by applying what is known about the dynamics of germs tangent to the identity. In [1.26] a particular family of 2-dimensional germs in  $\mathbb{C}^2$  with  $df_p = J_2$ , the canonical Jordan 2-by-2 matrix associated to the eigenvalue 1, is studied in detail, both in the real and in the complex setting, showing the existence of an open basin of attraction of the origin not predicted by previous results.

In [1.25], using a combination of techniques coming from differential geometry, algebraic geometry and complex analysis, a complete generalization of the classical Leau-Fatou flower theorem to two complex variables is proved: every germ tangent to the identity in two complex dimension having an isolated fixed

point admits parabolic curves (in [1.26] this results have been extended to two-dimensional germs with  $df_p = J_2$ ). The main new tool introduced here is an index theorem for holomorphic self-maps modeled on the Camacho-Sad index theorem for holomorphic foliations; see 14.2.3 below for far-reaching generalizations of this result.

In [1.28], written in collaboration with F. Tovena, we study examples of germs tangent to the identity in  $\mathbb{C}^3$  with parabolic curves showing a behavior not present in 2-dimensional germs.

Papers [2.14, 2.18, 2.19, 2.23] are surveys, of different depths, of the theory of local discrete holomorphic dynamical systems in one or several complex variables. In particular, [2.19] possibly is the most accessible and comprehensive introduction to the subject presently available.

One of the main open problems in local holomorphic dynamics consists in giving a complete description of the dynamics of a germ tangent to the identity in a full neighborhood of the fixed point. Paper [1.35], written in collaboration with F. Tovena, contains for the first time such a description for a substantial class of meaningful examples, obtained as time-1 maps of suitable homogeneous vector fields. This result is obtained combining results coming from two different theories, both interesting for their own sake. The first one is the theory of geodesics for meromorphic connections on Riemann surfaces (see 14.2.4 below); indeed, using the full force of the geometrical structure introduced in [1.29] we are able to prove that there is a  $\nu$ -to-one correspondence between real integral curves of a homogeneous vector field of degree  $\nu + 1$  in  $\mathbb{C}^n$  not contained in a characteristic line and geodesics for a suitable partial meromorphic connection on  $\mathbb{P}^{n-1}(\mathbb{C})$  defined along a singular foliation in Riemann surfaces. The second one is the theory of normal forms of singularities of meromorphic connections on Riemann surfaces (see 14.1.4 below), allowing us to give a description of the local behavior of geodesics (and hence of integral curves of homogeneous vector fields) nearby generic singularities. Combining this with results on the global asymptotic behavior of geodesics (Poincaré-Bendixson-like theorems) we get a wealth of information on the dynamics of homogeneous vector fields, allowing in particular to discover unexpected new phenomena and to give a complete description of the behavior in a substantial class of examples. [2.20] is a short survey of our results in this area.

[2.23] is a survey of the known results (up to July 2015) on the existence of parabolic curves in several complex variables.

*14.1.4. Normal forms.* One of the most powerful tool in the study of local dynamics is given by reduction to normal forms, the Poincaré-Dulac normal form being the most well known in this setting. The idea is to express a given germ in a simpler (normal) form by a change of variables of suitable smoothness; to classify the possible normal forms one can obtain; and then to study the dynamics of the normal forms.

Paper [2.15] describes all possible holomorphic normal forms of quadratic maps tangent to the identity in complex dimension 2, describing what was known about their dynamics and stating a few open problems.

Paper [2.16], written in collaboration with F. Tovena, describes a new technique for obtaining formal (i.e., with respect to change of coordinates given by not necessarily convergent power series) normal forms of germs tangent to the identity having a curve of fixed points, and provides a list of possible normal forms in dimension 2.

As mentioned above, [1.35], written in collaboration with F. Tovena, studies the normal forms of singularities of meromorphic connections on Riemann surfaces. It turns out that there are three different classes of singularities: apparent, Fuchsian (the generic case), and irregular. We obtain a complete classification of holomorphic normal forms for apparent and Fuchsian singularities; and a complete classification of formal normal forms for irregular singularities.

In [1.41], written in collaboration with J. Raissy, we describe a procedure, inspired by [2.16], producing formal normal forms simpler than the classical Poincaré-Dulac normal forms, and we apply it to classify 2-dimensional superattracting germs with a non-vanishing quadratic part.

*14.1.5. Other results.* The book [3.3] is an introduction to (real) hyperbolic dynamical systems, describing the main basic results from the topological, smooth and ergodic point of views. In particular, this monograph contains a complete proof of the stable manifold theorem for not necessarily invertible self-maps.

Paper [1.30], written in collaboration with F. Bracci, generalizes to hyperbolic manifolds of any dimension classical one-dimensional results due to Ritt and Heins on the holomorphic dependence on the map of the unique fixed point of a self-map having relatively compact image.

[1.34], written in collaboration with F. Bracci, M.D. Contreras and S. Díaz-Madrigal, summarizes the history and main applications of Loewner's differential equation.

[1.38], as the title indicates, is a collection of open problems in local discrete holomorphic dynamics.

In [1.42], written in collaboration with A. Abbondandolo and P. Majer, we give a sufficient condition for the abstract basin of attraction of a sequence of holomorphic self-maps of balls in  $\mathbb{C}^n$  to be biholomorphic to  $\mathbb{C}^n$ . As a consequence, we obtain a sufficient condition for the stable manifold of a point in a compact hyperbolic invariant subset of a complex manifold to be biholomorphic to a complex Euclidean space, generalizing previous results by Jonsson-Varolin and Peters.

## 14.2. Complex differential geometry.

*14.2.1. Hermitian symmetric spaces.* In [1.1, 1. 2] is presented a construction of the group of biholomorphisms of the four families of classical bounded symmetric domains, simpler than the previous constructions due to Siegel, Klingen and Morita.

In [1.4] it is described the fine structure of the space of orbits of an irreducible Hermitian symmetric space of non compact type under the action of the isotropy group of one point, generalizing results by Wolf, Korányi, Harris and others.

[2.4] contains a complete description of the complex geodesic (see 14.3.1) of non-compact Hermitian symmetric spaces, based upon the techniques introduced in [1.4].

*14.2.2. Complex Finsler metrics.* During the '90s, S.-S. Chern and his collaborators started a deep study of the global differential geometry of real Finsler manifolds; in the same period, G. Patrizio and myself started a similar study of the global differential geometry of complex Finsler manifolds. Our motivations came from the study of the Kobayashi metric (see 14.3.1), which is indeed a complex Finsler metric (in general only upper semicontinuous, but smooth in some important cases — e.g., in strongly convex domains, as proved by Lempert).

Our first work in this subject is [1.13] (see also [2.5]), where we describe a differential characterization of complex geodesics (i.e., of isometric holomorphic maps from the unit disk  $\Delta \subset \mathbb{C}$  endowed with the Poincaré metric to the complex Finsler manifold) for sufficiently regular complex Finsler metrics. When applied to the Kobayashi metric, this result allowed us to define a sort of exponential mapping, leading to a new invariant characterization of strongly pseudoconvex circular domains.

Our study started in earnest with paper [1.18], where using results from [1.13] we found a differential geometric characterization of complex Finsler metrics admitting complex geodesics: they are the complex Finsler metrics having constant negative holomorphic sectional curvature and satisfying a suitable Kähler condition. With similar techniques in [2.6] we showed that Kähler Finsler manifolds with constant positive (respectively, zero) holomorphic sectional curvature are foliated by isometric immersions of  $\mathbb{P}^1$  (respectively,  $\mathbb{C}$ ). Later on, in [1.20] we were able to give a complete classification of Kähler Finsler manifolds with constant nonnegative holomorphic sectional curvature, and we also further clarified the structure of Kähler Finsler manifolds with constant negative holomorphic sectional curvature. To appreciate the scope of these results it should be noticed that it does not exist a classification of real Finsler manifolds having constant sectional curvature.

To conduct our studies we had to build essentially from scratch the foundations of global differential geometry of complex Finsler manifolds; our approach is described in detail in the monograph [3.2] (partly anticipated in [2.8]). We start by giving a presentation, inspired by the work of Chern, of the global differential geometry of real Finsler manifolds, from the basic definitions to a proof of the Cartan-Hadamard theorem for Finsler metrics, using the Cartan connection and the first and second variation formulas. We then describe (as far as we know for the first time) the foundations of the global differential geometry of complex Finsler manifolds. The main tool here is the Chern-Finsler connection, used to express in complex terms the variation formulas, to introduce several notions of Kählerianity, and to study the holomorphic sectional curvature. In the last part, we describe (mainly following [1.13, 1.18] and [2.6]; paper [1.20] was written after the completion of this monograph) the geometrical structure of Kähler-Finsler manifolds with constant negative holomorphic sectional curvature; the construction of exhaustions satisfying the complex Monge-Ampère equation in Kähler-Finsler manifolds with constant nonpositive holomorphic sectional curvature; and the proof that the only simply connected manifold admitting a complete Kähler-Finsler metric with constant vanishing holomorphic sectional curvature is  $\mathbb{C}^n$ . After twenty years from its publication this monograph still is the main reference for the foundations of complex Finsler geometry.

In [2.7] the techniques we developed are applied to give a new characterization of tube domains; [2.9] describes how these techniques can be used to find pluricomplex Green functions with given pole; and [1.19] is devoted to a comparison of the three main connections used in the study of real Finsler metrics. Finally,

the preface [2.10], written with T. Aikou and G. Patrizio, is a short survey of the history and main results of complex Finsler geometry.

*14.2.3. Index theorems.* In the literature there were a few index theorems concerning holomorphic self-maps having only isolated fixed points, the holomorphic Lefschetz index theorem being the most well-known among them, but none concerning holomorphic self-maps having a positive dimensional fixed points set. In the ground-breaking paper [1.29], written in collaboration with F. Bracci and F. Tovena, we proved three different index theorems for holomorphic self-maps having a positive dimensional, possibly singular, fixed point set; these theorems can be considered a discrete analogue of the Baum-Bott, Camacho-Sad and Lehmann-Suwa-Khanedani index theorems for foliations (and a far-reaching extension of the 2-dimensional index theorem proved in [1.25] mentioned in 14.1.3). Furthermore, and possibly more importantly, in [1.29] it is also shown that the simple fact of being the fixed point set of a holomorphic self-map yields a rich geometric structure — somehow encoding the dynamics of the map in an infinitesimal neighborhood of the fixed point set  $S$  — including a singular holomorphic foliation in Riemann surfaces of  $S$ , a canonical morphism between suitably defined vector bundles on  $S$ , and a meromorphic connection along the leaves of the foliation. This geometric structure has been later applied in [1.35] to the study of the dynamics of homogeneous vector fields; see 14.1.3.

The similarities between our index theorems for self-maps and the index theorems valid for singular holomorphic foliations have been explained in [1.31], written in collaboration with F. Bracci and F. Tovena, where we described a very general procedure for proving index theorems depending only on the existence of suitable partial meromorphic connections, and we showed how to recover all the previously mentioned (and a couple of new ones) index theorems, both for self-maps and for foliations, making use of this procedure. Paper [2.17] is a short presentation of our approach for the case of Camacho-Sad-like index theorems, and paper [1.40], written in collaboration with F. Bracci, T. Suwa and F. Tovena, applies a similar procedure to the Atiyah characteristic class associated to any  $(1, 0)$ -connection on a complex vector bundle (see also 14.2.4).

Paper [2.21] shows how it is possible to deduce from our index theorems a few index theorems for meromorphic self-maps of the projective space, generalizing results obtained by Ueda and others for holomorphic self-maps having isolated fixed points only.

*14.2.4. Meromorphic connections.* Meromorphic connections on line bundles over Riemann surfaces are a classical subject, related to the study of linear differential systems. In paper [1.35], written in collaboration with F. Tovena and motivated by dynamical problems (see 14.1.3 above), we study them from a novel point of view. We introduce the notion of geodesic for a meromorphic connection as real curves having parallel tangent vector in a suitable sense, and we study in detail the asymptotic behavior of such geodesics. In particular, using the Gauss-Bonnet theorem we link the behavior of the geodesics with the residues of the meromorphic connection, and we prove a Poincaré-Bendixson theorem of the structure of the  $\omega$ -limit of a geodesic for a meromorphic connection on  $\mathbb{P}^1(\mathbb{C})$ . In paper [1.45], written in collaboration with F. Bianchi, we prove a similar theorem for meromorphic connections defined on a compact Riemann surface, and we study in detail the geodesics for holomorphic connections on tori.

In [1.40], written in collaboration with F. Bracci, T. Suwa and F. Tovena, using a Chern-Weil approach to Čech-Dolbeault cohomology we describe the Atiyah characteristic class measuring the obstruction to the existence of holomorphic connections on a complex vector bundle. We then describe a localization principle showing how it is possible to infer index theorems from vanishing theorems for the Atiyah class, following an approach analogous to the one introduced in [1.31].

*14.2.5. Other results.* [1.5] is devoted to the study of annular bundles, that is fiber bundles with complex plane annuli as fibers, and of holomorphic maps between them, generalizing the classical theory of Landau and Ossermann on holomorphic functions between plane annuli.

[1.14], written in collaboration with L. Geatti, is devoted to the classification of topologically trivial hyperbolic Stein manifolds  $X$  where a compact connected Lie group  $K$  acts holomorphically so that the orbit space  $X/K$  has real dimension 2. It turns out that all such manifolds can be obtained starting from a pseudoconvex Reinhardt domain in  $\mathbb{C}^2$  containing the origin and from the isotropy group of a point in bounded symmetric domain of rank 2 in  $\mathbb{C}^n$ .

[1.15], written in collaboration with P. Heinzner, contains a construction of Lie groups acting holomorphically without fixed points on topologically contractible domains; this result also provides an interesting

dynamical counterexample (see 14.1.1).

In our study of index theorems [1.29, 1.31] we identified two conditions on the way a complex submanifold can be embedded in the ambient manifold, conditions expressible in terms of infinitesimal neighborhoods. In [1.33], written with F. Bracci and F. Tovena, we showed that these two conditions are just the first in a whole family of conditions, expressing relationships between the embedding in the ambient manifold and the embedding as the zero section in the normal bundle, in the spirit of Grauert and Griffiths.

### **14.3. Geometric function theory and geometric analysis of several complex variables.**

*14.3.1. Invariant metrics and distances.* Invariant metrics and distances, introduced by Carathéodory, Kobayashi and others, are a very important tool for the study of geometric function theory and geometric analysis in several complex variables, because they give a geometric way for representing analytic properties of holomorphic maps. In particular, as shown by Vesentini, Lempert and others, the study of complex geodesics, that is of holomorphic maps from the unit disk  $\Delta \subset \mathbb{C}$  into a complex manifold  $X$  which are isometries with respect to the Poincaré distance of  $\Delta$  and the Kobayashi (or Carathéodory) distance of  $X$ , can give a lot of information on the geometry and analysis of the manifold  $X$ .

The paper [1.3] contains two related results: an important estimate on the boundary behavior of the Kobayashi distance in bounded strongly pseudoconvex domains (that has been later extended to other domains by Krantz, Rosay, Forstneric, Catlin and many others), and the proof that complex geodesics in strongly pseudoconvex domains extend  $1/2$ -Hölder to the boundary, generalizing a result obtained by Lempert in strongly convex domains.

As mentioned above, [2.4] contains the description of complex geodesics in non-compact Hermitian symmetric spaces.

In [1.13] (see also [2.6]), written in collaboration with G. Patrizio, we studied complex geodesics in circular domains using differential geometric techniques, obtaining a new invariant characterization of strongly pseudoconvex circular domains.

[1.16] contains a characterization of Kobayashi hyperbolic manifolds (i.e., of complex manifolds whose Kobayashi pseudodistance is a true distance), clearly explaining the relationship between such manifolds and taut manifolds.

Practically by definition, a biholomorphism is an isometry for any invariant metric or distance; a classical problem in this area consists in discovering when the inverse implication holds. In [1.32], written in collaboration with J.-P. Vigué, we show that, under suitable hypotheses on the Banach spaces involved, a holomorphic map between the unit balls of two (possibly infinite dimensional) Banach spaces which is an isometry for the Carathéodory metric at one point necessarily is a biholomorphism with its image, and the image is a (necessarily smooth) holomorphic retract of the codomain.

*14.3.2. Boundary behavior of holomorphic maps.* One of the topics where the analysis in several complex variables differs from the analysis in one complex variable is the topic of boundary behavior of holomorphic maps; in several variables one obtains results that are stronger than expected compared to the one-variable analogues, and new subtle phenomena appear.

The classical one-variable Lindelöf principle says that for a bounded holomorphic function defined on the unit disk  $\Delta \subset \mathbb{C}$  the existence of the limit along a curve ending at a boundary point implies the existence of the non-tangential limit at that point. Čirka and Cima and Krantz have found versions of this principle in several complex variables, involving a notion of restricted  $K$ -limit (or hypoadmissible limit) which is stronger than non-tangential limit; paper [1.9], combining the ideas of these authors, introduces a new family of Lindelöf principles for not necessarily bounded holomorphic functions. Furthermore, together with the theory of invariant distances and complex geodesics (see 14.3.1), these new Lindelöf principles are used to extend to bounded strongly convex domains the classical theorem of Julia-Wolff-Carathéodory on the angular limit of the derivative of a holomorphic map, generalizing results obtained by Rudin in the unit ball of  $\mathbb{C}^n$ . These results have been later generalized to strongly pseudoconvex domain (in [2.3]), to polydisks (in [1.23]), and to convex domains of finite type (in [1.27], written in collaboration with R. Tauraso), showing the wide applicability of the techniques introduced. Surveys [2.11], written in collaboration with R. Tauraso, and [2.13] describe the theory up to 2004, proving a few more new Julia-Wolff-Carathéodory theorems along the way.

In paper [1.44], written in collaboration with J. Raissy, we study the boundary behavior of infinitesimal generators of one-parameter semigroups in the unit ball of  $\mathbb{C}^n$ ; we obtain a version of a Julia-Wolff-

Carathéodory theorem, extending and making more precise previous results by Bracci and Shoikhet, showcasing intriguing differences with respect to similar results for holomorphic maps.

*14.3.3. Carleson measures and Toeplitz operators.* Carleson measures for Hardy spaces in the unit disk were introduced in the '60s by Carleson in his celebrated solution of the corona problem; since then they have become an interesting subject of study on their own.

Carleson measures are defined in function theoretical terms: a measure  $\mu$  is a Carleson measure for a Banach space of holomorphic functions  $A$  and a  $p \geq 1$  if  $A$  embeds continuously in  $L^p(\mu)$  (and it is a vanishing Carleson measure if this embedding is compact). In paper [1.36], written with A. Saracco, we give a geometrical characterization of Carleson measures of Bergman spaces of strongly pseudoconvex domains in terms of the behavior of the measure of balls for the intrinsic Kobayashi distance (see 14.3.1) and in terms of the boundary behavior of the Berezin transform of the measure, generalizing results due to Luecking, Duren, Weir and others for the unit ball of  $\mathbb{C}^n$ .

In [1.39], written in collaboration with J. Raissy and A. Saracco, we first of all obtain a geometric characterization of Carleson measures and vanishing Carleson measures of weighted Bergman spaces in strongly pseudoconvex domains. Using this characterization we are able to perform a very detailed study of the mapping properties of Toeplitz operators having as symbol a measure; in particular, we are able to prove a conjecture due to Čučkovic and McNeal. The main tools used are the intrinsic Kobayashi geometry of strongly pseudoconvex domains, and a precise estimate of the weighted  $L^p$  norm of the Bergman kernel.

[1.47] is a survey paper describing, among other things, the results obtained in [1.36] and [1.39].

*14.3.4. Other results.*

In [1.8] complex geodesics (see 14.3.1) and the theory of holomorphic dynamical systems (see 14.1.1) are used to prove that a commuting family of holomorphic self-maps continuous up to the boundary of a bounded strongly convex domain admits a common fixed point, generalizing a result proved by Shields in one variable. In [1.10], written with J.-P. Vigué, the theorem is extended to all bounded convex domains.

A completely explicit algorithm to determine whether a linear self-map of a finite dimensional (real or complex) vector space is diagonalizable without computing its eigenvalues is described in [1.21].

In [1.22, 2.12] (written in collaboration with G. Patrizio) our results on complex geodesics and complex Finsler metrics (see 14.3.1 and 14.2.2) are applied to the case of (possibly infinite dimensional) Teichmüller spaces, generalizing Royden's theorem characterizing biholomorphisms of Teichmüller spaces as holomorphic maps which are isometries for the Teichmüller metric at one point. We also describe some parallelisms between the geometry of Teichmüller spaces and the geometry of convex domains.

[1.46] is a biochemistry paper where I provided the mathematical fitting of the experimental data.

## 15. Textbooks, popularizing and outreach work

In this section I shall briefly describe the content of my textbooks and of the popularizing papers I wrote. Again, numbers in square brackets refer to the list of publications.

**15.1. Textbooks.** The (pretty successful) books [4.1, 4.3, 4.5] (the latter in collaboration with C. de Fabritiis) are textbooks for a first year undergraduate course in linear algebra and basic geometry for mathematicians, physicists, computer scientists and engineers. The topics covered include linear systems, finite dimensional vector spaces, linear maps, determinants, eigenvalues and eigenvectors, affine and euclidean geometry, bilinear and quadratic forms; the exposition tries to make explicit the motivations behind the main definitions and results, with the aim of bringing the student to a full and personal understanding of the subject, also via several examples completely worked out. [4.2] (in collaboration with C. de Fabritiis) is a collection of exercises useful as a complement to [4.1, 4.3, 4.5].

[4.4] is the Italian edition of an American *Calculus* book, with a new chapter on linear algebra written by me.

[4.6], written with F. Tovena, is a textbook on the differential geometry of curves and surfaces in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ . The topics cover the local theory of curves, a few fundamental theorems in the global theory of curves (including a complete proof of the Jordan curve theorem), the local theory of surfaces (including curvatures and a complete proof of the fundamental theorem of the local theory of surfaces), the theory of geodesics, and a few fundamental results in the global theory of surfaces (including a complete proof of the Gauss-Bonnet theorem, and the classification of surfaces with constant Gaussian curvature). The book has been successful enough to deserve an English translation [4.9].

[4.7] is a (very successful) textbook for a first year undergraduate course in calculus and statistics for students in life and earth sciences (including biology, biotechnology, natural sciences, geology, etc.). The topics covered include basic probability theory (both discrete and continuous), basic statistics (including the least square methods and other interpolation techniques), calculus of one real variable, an introduction to differential equations, and basic linear algebra. The book is written with the aim of revealing to the students the usefulness of mathematics in understanding, building and discussing models of natural phenomena, and contains more than one hundred (slightly simplified but still) real-life examples worked out in detail. In this way the students can appreciate the why and how of mathematical tools, without renouncing to the usual mathematical rigor, and are led to a firmer understanding of the subject.

[4.8], written with F. Tovena, is a fairly comprehensive introduction to differential geometry of smooth manifolds. The topics covered include an introduction to multilinear algebra, smooth manifolds (including a proof of Whitney's embedding theorem), Lie groups, vector bundles (including a proof of Frobenius theorem), Lie algebras, differential forms (including a proof of Stokes theorem), de Rham cohomology (including proofs of Poincaré duality, Künneth theorem and de Rham theorem), sheaves and sheaf cohomology, connections, Riemannian metrics, geodesics (including proofs of the Whitehead theorem on geodesically convex neighborhoods and of the Hopf-Rinow theorem), and curvature (including proofs of the Cartan-Hadamard, Bonnet-Myers, Weinstein and Synge theorems).

**15.2. Popularizing and outreach papers.** [5.1, 5.2] deal with the representation of mathematics in comic books; [5.5] describes a possible comic book story loosely inspired by Galois' life.

[5.3] is an introduction to chaotic dynamical systems written for a general public.

[5.4] is a popularizing exposition of the Radon transform and the Banach-Tarski paradox.

[5.6, 5.7] are two short pieces on popularizing science in Italy.

[5.8] talks about the minimizing properties of the Fibonacci sequence in sunflowers and other natural phenomena.

[5.9] discusses Hardy's autobiography *A Mathematician's apology*.

[5.10] is an introduction to local discrete holomorphic dynamical systems in one complex variables, written for a general mathematical public.

[5.11, 5.17] are book reviews.

[5.12] is an encyclopedia piece discussing the most important mathematical results on dynamical systems published in the first ten years of the 21st century.

[5.13] discusses what the theory of chaotic dynamical systems can say about the hypothesis of "invisible hand" in economy suggested by Adam Smith.

[5.14] presents the history of the study of the stability of the solar system, from the ancient Greeks to recent results due to Laskar.

[5.15] presents an intriguing and unexpected relationship found by Ghys between the Lorentz chaotic attractor and the topological theory of knots.

[5.16] is a presentation of the work of John Milnor on differential topology, and in particular of the work leading to the discovery of exotic spheres.

[5.18] discusses the history of Newton's methods for finding roots of polynomials, up to the recent work of Hubbard and collaborators,

[5.19] is a short autobiographical piece on why I have become a mathematician.

[5.20] is a presentation to fractal art, mainly composed by interviewing fractal artists.

**15.3. Editing of books.** [6.1] contains the texts of the talks given in the conference *Perché Nobel? 2007*, explaining to a general public the content of the work of the winners of the 2007 Nobel and Abel prizes.

[6.2] contains the text of four mini courses describing the foundations of the theory of singular holomorphic foliations.

[6.3] is the volume of proceedings of the conference *Matematica e Cultura 2014* held in Venice in March 2014.



# PUBLICATIONS

Marco Abate

## 1. Research papers

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